SOME RESULTS ON EVEN SUM GRAPHS

ABSTRACT

Chinju Krishna K interpolated the concept of Even Sum Graphs (ESG). A graph with labeling of its vertices using distinct non – negative integers in which the sum of any two points of the labels which are adjacent is also a label of a point in that graph is said to be an ESG. The notion of even sum number for G is $\gamma(G)$ & is defined as the number of isolated points needed to make G, an ESG and that number should be the *minimum.*

Keywords : *path, cycle, flower graph, ladder graph, isolated vertex.*

AMS Classification *: 05C70, 05C05,05C75, 05C78*

Introduction:

The assignment of labels to edges and or vertices of a graph is traditionally represented by integers is called labeling. Frank Harary interpolated Sum Graphs(SG) & Integral Sum Graphs(ISG). A graph with labeling of its points using distinct positive integers with the intention that the sum of any two points in a line are adjacent & is also a label of the vertex in that graph is called a SG. $\sigma(G)$ means that how many isolated points needed to make the graph, a SG and that number should be the minimum. ISG are also defined in the same manner with distinct integers. $\xi(G)$ denotes the number of isolated points needed to make a graph, an ISG and that number should be the minimum. The properties of SG & ISG was studied by many authors[1, 9, 10, 11] In this article, we enquires on various ESG [2,3,4]. We refer [5, 6] for all basic ideas. A $(n \times m)$ - flower graph denoted by $f_{n \times m}$ has $n(m - 1)$ points and mn lines. We denote a graph obtained by attaching paths of lengths, 1, 2, …, n – 2, n - 1 respectively on both sides of each vertex of P_n by $P_n(P_1, 2P_2, 2P_3, ..., 2P_n)$. A ladder graph is given by the cartesian product of 2 paths, i.e, $P_2 \times P_n$ & one of which has only one edge, A ladder graph is denoted by L_n.

Primary Results:

Result : 3.1. $f_{n\times 3} \cup (n-1)K_1$ is an ESG.

Proof: Let $a_1, a_2, \ldots, a_n$ be the vertices of n – cycles of $f_{n\times 3}$ & let $\{b_i\}$ be the i^{th} set of vertices, $1 \le i \le n$ which form 3 cycles around the n -cycles in which 3- cycles intersects with n – cycles on a single edge. Then $E(f_{n\times 3} \cup (n-1)K_1) = \{a_ib_i / 1 \le i \le n\} \cup \{a_ia_{i+1}, a_ib_{i+1}, a_na_1, a_nb_1 / 1 \le i \le n-1\}$.

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Let $c_1, c_2, \ldots, c_{n-1}$ be the isolated vertices of $f_{n\times 3} \cup (n-1)K_1$. Now we defined $\eta : V(f_{n\times 3} \cup (n-1)K_1) \rightarrow 2Z^+ \cup \{0\}$ given by $\eta(a_1) = 0$; $\eta(a_2) = 2$; $\eta(a_3) = 4$; $\eta(a_i) = \eta(a_{i-1}) + \eta(a_{i-2})$, where $i = 4, \ldots, n$; $\eta(b_1) = \eta(a_{n-1}) + \eta(a_n)$; $\eta(b_2) = \eta(b_1) + \eta(a_n)$; $\eta(b_i) = \eta(a_{i-1}) + \eta(b_{i-1})$, where $i = 3, 4, \ldots, n$; $\eta(c_1) = \eta(a_n) + \eta(b_n)$; $\eta(c_i) = \eta(b_{i+1}) + \eta(a_{i-1})$, $2 \leq i \leq n-1$. Then we get distinct labels and for any edge ab in G, the condition for ESG holds. Thus $f_{n\times 3} \cup (n-1)K_1$ is an ESG.

Illustration: 3.2. An ESG of $f_{6\times 3} \cup 5K_1$ is shown in Figure 3.1.

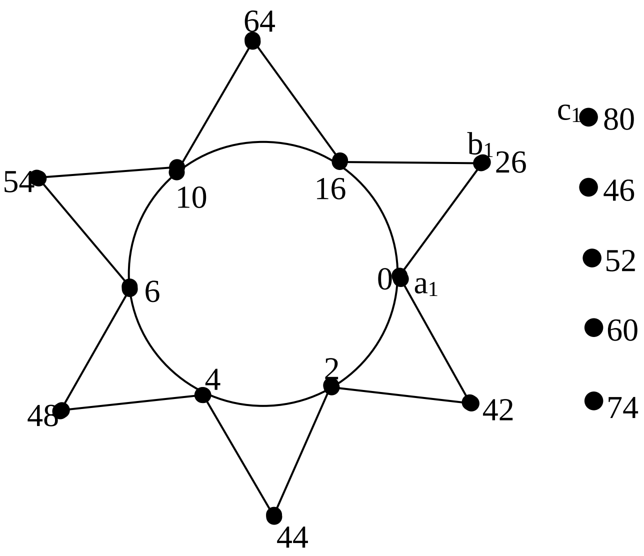

Figure: 3.1

Result: 3.3. $P_n(P_1, 2P_2, 2P_3, \ldots, 2P_n) \cup 2K_1$ is an ESG.

Proof: Let $a_{i,j}$, where $1 \leq i \leq n$ & $1 \leq j \leq 2i - 1$ be the vertex of $P_n(P_1, 2P_2, 2P_3, \ldots, 2P_n)$ and let b_1 and b_2 be the isolated vertices of G. Then $E(P_n(P_1, 2P_2, 2P_3, \ldots, 2P_n) \cup 2K_1) = \{a_{i,i}\, a_{i+1,i+1} / 1 \leq i \leq n-1\} \cup \{a_{i,j}\, a_{i,j+1} / 2 \leq i \leq n, 1 \leq j \leq 2i-2\}$. Now we defined a function $\eta : V(P_n(P_1, 2P_2, 2P_3, \ldots, 2P_n) \cup 2K_1) \rightarrow 2Z^+ \cup \{0\}$ given as $\eta(a_{1,1}) = 0$; $\eta(a_{2,2}) = 4$; $\eta(a_{2,1}) = 2$; $\eta(a_{i,1}) = \eta(a_{i-1,j}) + (a_{i-1,j+1})$, where $3 \leq i \leq 2i-1$, $j = i-2$; $\eta(a_{3,2}) = \eta(a_{3,1}) + 2$; $\eta(a_{i,2}) = \eta(a_{i-2,i-2}) + \eta(a_{i-1,i-1})$, where $4 \leq i \leq n$; $\eta(a_{i,j}) = \eta(a_{i,j-2}) + \eta(a_{i,j-1})$, $\eta(b_1) = \eta(a_{n,2n-2}) + \eta(a_{n,2n-1})$; $\eta(b_2) = \eta(a_{n,n}) + \eta(a_{n-1,n-1})$. Then the labels are distinct and for any edge ab, the condition for ESG holds. Thus $P_n(P_1, 2P_2, 2P_3, \ldots, 2P_n) \cup 2K_1$ is an ESG.

Illustration: 3.4. An ESG of $P_5(P_1, 2P_2, 2P_3, \ldots, 2P_5) \cup 2K_1$ is shown in the Figure 3.2.

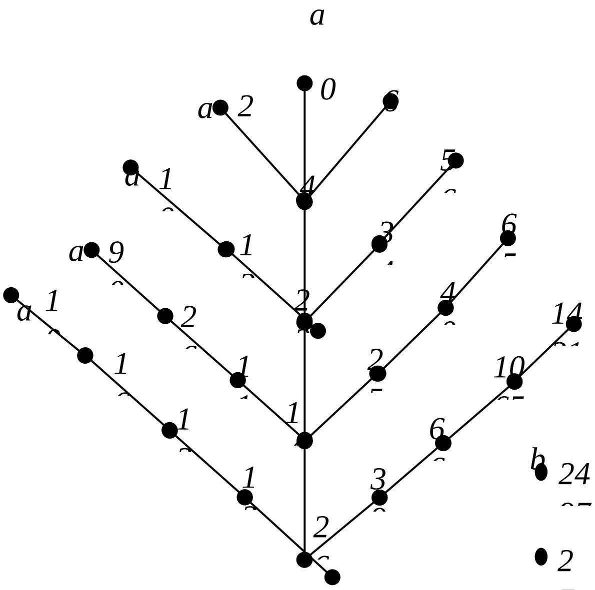

Figure: 3.2

Result: 3.5 $(L_n \, \overline{\odot K_2}) \cup (n-2)K_1$ is an ESG.

Proof: Let a_i, b_i where $1 \le i \le n$ be the points of two P_n graphs. Join a_i and b_i where $1 \le i \le n$. Then we obtained L_n . Join b_i with c_i and d_i where $1 \le i \le n$ and a_i with the vertices e_i and f_i where $1 \le i \le n$. Then we get $L_n \odot \overline{K_2}$ & $E((L_n \odot \overline{K_2}) = \{a_ie_n, a_if_n, b_ic_i, b_id_i / 1 \le i \le n\} \cup \{b_ib_{i+1}, a_ia_{i+1} / 1 \le i \le n-1\}$. Let $g_1, g_2, \ldots, c_{n-2}$ be the isolated vertices of G. Now let us define $\eta : V((L_n \odot K_2) \cup (n-2)K_1) \rightarrow 2Z^+ \cup \{0\}$ given by $\eta(a_2) = 2$; $\eta(a_1) = 0$; $\eta(a_3) = 4$; $\eta(a_i) = \eta(a_{i-1}) + \eta(a_{i-2})$, where $i = 4, \ldots, n$; $\eta(b_1) = \eta(a_{n-1}) + \eta(a_n)$; $\eta(b_2) = \eta(b_1) + 2$; $\eta(b_i) = \eta(b_{i-1}) + (b_{i-2})$, where $i = 3, \ldots, n$; $\eta(c_1) = \eta(b_{n-1}) + \eta(b_n)$; $\eta(c_i) = \eta(b_{i-1}) + \eta(d_{i-1})$, $i = 2, \ldots, n$; $\eta(d_i) = \eta(b_i) + \eta(c_i)$, $i = 1, \ldots, n$; $\eta(e_1) = \eta(b_n) + \eta(d_n)$; $\eta(e_i) = \eta(a_i) + \eta(b_i)$, where $i = 2, \ldots n$; $\eta(f_i) = (e_{i+1}) + \eta(a_{i+1})$, where $i = 1, 2, \ldots,$

Illustration: 3.6. An ESG of $(L_5 \odot \overline{K_2}) \cup 3K_1$ is shown in the Figure 3.3.

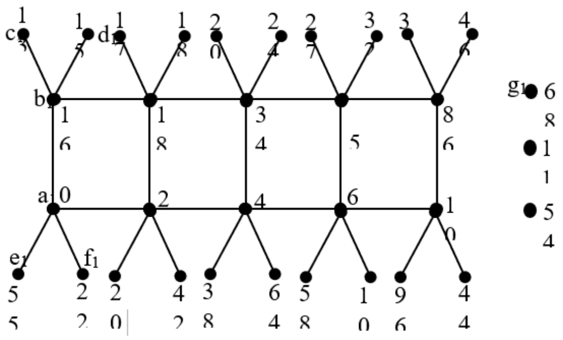

Figure: 3.3

Conclusion:

In this article, we constructed some ESG using isolated vertices & this article provides to obtain similar results on various types of ESG. It also helps to identify different labeling techniques

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