AN INTUITIONISTIC FUZZY MULTISET THEORETIC APPROACH IN MCDM PROBLEMS

ABSTRACT

In recent years Intuitionistic Fuzzy sets (IFSs) are beneficial in decision making problems. Since Intuitionistic Fuzzy Multisets (IFMSs) are the extension IFSs it has an outstanding impact in the field of decision making. We introduced a New Distance measure for IFMSs in this paper, and to demonstrate the effectiveness of the proposed distance measure, we applied it to Multi Criteria Decision Making Problems.

Key words:*Intuitionistic Fuzzy Sets (IFS), Intuitionistic Fuzzy Multi Sets (IFMS), Distance Measure, Tangent Function, Medical Diagnosis.*

1. Introduction

In classical set theory, set means a well-defined collection of objects. That is there is a rigid boundary for sets. So, we can distinguish the objects into two groups: Members in the set and members not in the set. For example, Let A be the set of all even numbers. We can clearly say that, any given number belong to A or not belong to A. So, in ordinary set theoretic view, we can divide the set of all numbers into two groups: Members in A and members not in A.

Now a days, uncertainty is an important and unavoidable factor in decision making. But we cannot give a proper definition to qualitative properties expressed in common dialects like intelligence, beauty etc using the aspects of classical sets.

Fuzzy sets, which L. A. Zadeh proposed, are a practical tool for dealing with hazy conceptions. In Fuzzy sets each member is characterized by a degree membership. Let X be the universal set. A fuzzy set M can be defined as

$M = \{(x, \mu_M(x)) / x \epsilon X\}$, where $\mu_M(x)$ is the degree of belongingness of x in M. Many research works had been done in Fuzzy Set Theory. Many concepts in Classical set theory had extended to Fuzzy set theory and found valuable applications in various real-life problems where classical sets failed.

JESWIN B. GEORGE
Research Scholar, Department of Mathematics, St. Thomas College, Palai, Kerala, India

SHINY JOSE
Professor, Department of Mathematics, St. George College, Aruvithura, Kerala, India.

The approach of intuitionistic fuzzy sets, which is regarded as an extension of L.

Zadeh's fuzzy sets, was first introduced by K.T. Attanassov in 1983. In Intuitionistic Fuzzy Sets each element is characterized by a degree of membership and a degree of non-membership. Since IFSs incorporates the degree of belongingness and degree of non-belongingness it is more suitable than Fuzzy Sets in decision making.

In classical set theory, repetition of elements is not allowed. If we ignore this restriction, i.e, repeated occurrence of elements is allowed then we get a new mathematical structure called Multisets. As a generalization of fuzzy sets, Yager put forward the Fuzzy Multisets. Later, T.K. Shinoj and Sunil Jacob John combined the two concepts: Intuitionistic Fuzzy Sets and Fuzzy Multisets and it paved the way to introducing the new concept called Intuitionistic Fuzzy Multisets (IFMSs). Also, they explained the practical applications of IFMSs. IFMSs as an extension of IFSs, has got all the properties of IFSs. P. Rajeswari and N. Uma extended various distance measures, similarity measures of IFSs for IFMSs.

In this study, we extend our existing Distance measure for IFSs to provide a new Distance measure for IFMSs. We presented a practical application in the area of medical diagnosis to demonstrate the effectiveness of the proposed Distance Measure.

2. Preliminaries

Definition 2.1[1]

Let X be a given set. An Intuitionistic Fuzzy Set A in X is given $A = \{(x, \mu_A(x), \nu_A(x))/ x \epsilon X\}$, where $\mu_A, \nu_A: X \to [0,1]$, and $0 \leq \mu_A(x)+ \nu_A(x) \leq 1$. $\mu_A(x)$ is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of non-membership of x in A. For each $x \epsilon X$, $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$ is called the degree of hesitation.

Definition 2.2[12]

Let X be a nonempty set. A Fuzzy Multiset (FMS) A drawn from X is characterised by a function, 'count membership' of A denoted by CM_A such that $CM_A: X \ 7\!\to Q$ where Q is the set of all crisp multisets drawn from the unit interval [0,1]. Then for any $x \epsilon X$, the value $CM_A(x)$ is a crisp multiset drawn from [0, 1]. For each $x \epsilon X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu^1{}_A(x), \mu^2{}_A(x), ..., \mu^P{}_A(x))$ where $\mu^1{}_A(x) \geq \mu^2{}_A(x) \geq, ..., \geq \mu^P{}_A(x)$.

Definition 2.3[9]

Let X be a nonempty set. An Intuitionistic Fuzzy Multiset A denoted by IFMS drawn from X is characterised by two functions :'Count membership' of A (CM_A) and 'count non-membership' of A(CN_A) given respectively by $CM_A : X \to 7\ Q$ and $CN_A : X \to 7\ Q$ where Q is the set of all crisp multisets drawn from the unit interval [0, 1], such that for each $x \epsilon X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CM_A(x)$ which is denoted by $(\mu^1{}_A(x), \mu^2{}_A(x), \ldots, \mu^P{}_A(x))$ where $\mu^1{}_A(x) \geq \mu^2{}_A(x) \geq, \ldots, \geq \mu^P{}_A(x)$ and the corresponding non-membership sequence will be denoted by $(\nu^1{}_A(x), \nu^2{}_A(x), \ldots, \nu^P{}_A(x))$ such that $0 \leq \mu^i{}_A(x) + \nu^i{}_A(x) \leq 1$ for every $x \epsilon X$ and i = 1, 2, . . . , p. An IFMS is denoted by

A = $\{< x: (\mu^1{}_A(x), \mu^2{}_A(x), \ldots, \mu^P{}_A(x)), (\nu^1{}_A(x), \nu^2{}_A(x), \ldots, \nu^P{}_A(x)) >: x \epsilon X\}$.

Definition 2.4[9]

Length of an element x in an IFMS A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu^i{}_A(x) + \nu^i{}_A(x) \leq 1$ and itis denoted by L(x : A). That is L(x : A) = $|CM_A(x)| = |CN_A(x)|$.

Definition 2.5[9]

If A and B are IFMSs drawn from X then L(x : A, B) = Max{L(x : A), L(x : B)} We can use the notation L(x) for L(x : A, B).

Definition 2.6[11]

A real-valued function $D: IFS(X) \times IFS(X) \to [0,1]$ is called a distance measure on IFS(X), if it satisfies the axiomatic requirements:

1. $0 \leq D(A, B) \leq 1$
2. D (A, B) = 1 if and only if A = B
3. D (A, B) = D (B, A)
4. $D(A, B) \leq D(A, C)$ and $D(B, C) \leq D(A, C)$ if $A \subseteq B \subseteq C$.

Definition 2.7[5]

Let $A = (\mu_A(x_i), \nu_A(x_i), \pi_A(x_i))$ and $B = (\mu_B(x_i), \nu_B(x_i), \pi_B(x_i))$ be two Intuitionistic Fuzzy Sets. Then the tangent inverse distance is defined as

$$T_{IFS}(A,B) = \frac{1}{2(n+1)} \sum_{i=1}^{n} [\tan^{-1}[d_i]]$$

where $d_i = |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|$.

Hence, $D_{IFS}(A, B) \leq D_{IFS}(A, C)$ and $D_{IFS}(B, C) \leq D_{IFS}(A, C)$.

3. Novel Distance Measure for IFMS

Definition 3.1

The new distance measure for IFMS based on tangent inverse function with three parameters membership, non-membership and hesitation function is

$$D_{IFS}(A,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}[\frac{1}{2(n+1)}\sum_{i=1}^{n} tan^{-1}(|\mu_A^j(x_i) - \mu_B^j(x_i)| + |\nu_A^j(x_i) - \nu_B^j(x_i)| + |\pi_A^j(x_i) - \pi_B^j(x_i)|)]$$

Definition 3.2

The defined new distance measure $D_{IFS}(A, B)$ between IFMS A and B satisfies the following properties

P1. $0 \leq D_{IFS}(A, B) \leq 1$

P2. $D_{IFS}(A, B) = 0$ if and only if A = B

P3. $D_{IFS}(A, B) = D_{IFS}(B, A)$

P4. If $A \subseteq B \subseteq C$, then $D_{IFS}(A, B) \leq D_{IFS}(A, C)$ and $D_{IFS}(B, C) \leq D_{IFS}(A, C)$

Proof:

***P*1**. $0 \le D_{IFS}(A, B) \le 1$.

Since $|\mu_A^j(x_i) - \mu_B^j(x_i)|$, $|\nu_A^j(x_i) - \nu_B^j(x_i)|$, $|\pi_A^j(x_i) - \pi_B^j(x_i)| \ge 0$ we get $\sum_{i=1}^{n} tan^{-1}(|\mu_A^j(x_i) - \mu_B^j(x_i)| + |\nu_A^j(x_i) - \nu_B^j(x_i)| + |\pi_A^j(x_i) - \pi_B^j(x_i)|)$.

Therefore $0 \le D_{IFS}(A, B) \le 1$.

P2. $D_{IFS}(A, B) \Leftrightarrow A = B$

$D_{IFS}(A,B) = 0$

$$\Leftrightarrow \sum_{j=1}^{\eta}\left[\frac{1}{2(n+1)}\sum_{i=1}^{n} tan^{-1}\left(|\mu_A^j(x_i) - \mu_B^j(x_i)| + |\nu_A^j(x_i) - \nu_B^j(x_i)| + |\pi_A^j(x_i) - \pi_B^j(x_i)|\right)\right] = 0$$

$$\Leftrightarrow \sum_{j=1}^{\eta}\left[\frac{1}{2(n+1)}\sum_{i=1}^{n} tan^{-1}\left(|\mu_A^j(x_i) - \mu_B^j(x_i)| + |\nu_A^j(x_i) - \nu_B^j(x_i)| + |\pi_A^j(x_i) - \pi_B^j(x_i)|\right)\right] = 0$$

$$\Leftrightarrow \sum_{i=1}^{n} tan^{-1}\left(|\mu_A^j(x_i) - \mu_B^j(x_i)| + |\nu_A^j(x_i) - \nu_B^j(x_i)| + |\pi_A^j(x_i) - \pi_B^j(x_i)|\right) = 0$$

$\Leftrightarrow \tan^{-1}(|\mu^j{}_A(x_i) - \mu^j{}_B(x_i)| + |\nu_A{}^j(x_i) - \nu_B{}^j(x_i)| + |\pi_A{}^j(x_i) - \pi_B{}^j(x_i)|) = 0$ for every i, j

$\Leftrightarrow (|\mu^j{}_A(x_i) - \mu^j{}_B(x_i)| + |\nu_A{}^j(x_i) - \nu_B{}^j(x_i)| + |\pi_A{}^j(x_i) - \pi_B{}^j(x_i)|) = 0$ for every i, j

$\Longleftrightarrow \mu_A^j(x_i) = \mu_B^j(x_i),\ \nu_A^j(x_i) = \nu_B^j(x_i)$ and $\pi_A^j(x_i) = \pi_B^j(x_i)$

$\Leftrightarrow A = B$

P3. $D_{IFS}(A,B)$

$$= \frac{1}{\eta}\sum_{j=1}^{\eta}[\frac{1}{2(n+1)}\sum_{i=1}^{n} \tan^{-1}(|\mu_A^j(x_i) - \mu_B^j(x_i)| + |\nu_A^j(x_i) - \nu_B^j(x_i)| + |\pi_A^j(x_i) - \pi_B^j(x_i)|)]$$

$= \frac{1}{\eta}\sum_{j=1}^{\eta}[\frac{1}{2(n+1)}\sum_{i=1}^{n} \tan^{-1}(|\mu_B^j(x_i) - \mu_A^j(x_i)| + |\nu_B^j(x_i) - \nu_A^j(x_i)| + |\pi_B^j(x_i) - \pi_A^j(x_i)|)]$

$= D_{IFS}(B,A)$.

***P*4**. If $A \subseteq B \subseteq C$, then

$\mu^j{}_A(x_i) \le \mu^j{}_B(x_i) \le \mu^j{}_C(x_i)$

$\nu^j{}_A(x_i) \ge \nu^j{}_B(x_i) \ge \nu^j{}_C(x_i)$

$\pi^j{}_A(x_i) \ge \pi^j{}_B(x_i) \ge \pi^j{}_C(x_i)$

Now,

$\mu^j{}_B(x_i) \le \mu^j{}_C(x_i) \Rightarrow |\mu^j{}_A(x_i) - \mu^j{}_B(x_i)| \le |\mu^j{}_A(x_i) - \mu^j{}_C(x_i)|$

$\mu^j{}_A(x_i) \le \mu^j{}_B(x_i) \Rightarrow |\mu^j{}_B(x_i) - \mu^j{}_C(x_i)| \le |\mu^j{}_A(x_i) - \mu^j{}_C(x_i)|$

$\nu^j{}_B(x_i) \ge \nu^j{}_C(x_i) \Rightarrow |\nu^j{}_A(x_i) - \nu^j{}_B(x_i)| \le |\nu^j{}_A(x_i) - \nu^j{}_C(x_i)|$

$\mu^j{}_A(x_i) \ge \mu^j{}_B(x_i) \Rightarrow |\nu^j{}_B(x_i) - \nu^j{}_C(x_i)| \le |\nu^j{}_A(x_i) - \nu^j{}_C(x_i)|$

Similarly,

$|\pi^j{}_A(x_i) - \pi^j{}_B(x_i)| \le |\pi^j{}_A(x_i) - \pi^j{}_C(x_i)|$

$$|\pi^j_B(x_i) - \pi^j_B(x_i)| \leq |\pi^j_A(x_i) - \pi^j_C(x_i)|$$

Hence, $D_{IFS}(A, B) \leq D_{IFS}(A, C)$ and $D_{IFS}(B, C) \leq D_{IFS}(A, C)$

4. Illustration

We will use a case study of illnesses that commonly affect children in this instance. Today, it is more difficult to group up various sets of symptoms under the banner of a single illness. The suggested distance between children versus symptoms and diseases vs symptoms helps with clinical diagnosis.

Let $C = \{C_1, C_2, C_3\}$ be a set of children. $D = \{$Viral fever, Throat problem, Chickenpox, Skin problem, Mumps$\}$ be the set of diseases and

$S = \{$Temperature, Headache, Throat pain, Muscle ache, Spots$\}$ be the set of symptoms.

Some diseases are listed in Table 1 together with their symptoms as intuitionistic fuzzy set values. Also, the diseases possess the same symptoms, although in varying degrees. Three numbers are used to describe each symptom: membership μ, non-membership v and hesitation margin π.

Table 1. symptom Vs Diseases

Relation1	Viral fever	Throat problem	Chickenpox	Skin problem	Mumps
Temperature	(0.8,0.2,0)	(0.7,0.1,0.2)	(0.2,0.6,0.2)	(0.5,0.3,0.2)	(0.2,0.5,0.3)
Headache	(0.2,0.5,0.3)	(0.3,0.7,0)	(0.8,0.1,0.1)	(0.4,0.3,0.3)	(0.1,0.7,0.2)
Throat Pain	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.0,0.6,0.4)	(0.2,0.7,0.1)	(0.0,0.5,0.5)
Muscle Ache	(0.3,0.6,0.1)	(0.5,0.4,0.1)	(0.3,0.4,0.3)	(0.7,0.2,0.1)	(0.3,0.4,0.3)
Spot	(0.2,0.5,0.3)	(0.1,0.7,0.2)	(0.8,0.1,0.1)	(0.6,0.1,0.3)	(0.5,0.1,0.4)

The objective is to make a proper diagnosis for each child. Let the samples be taken at three different timings in a day (morning, noon, night). After the samples obtained, we get a supposed medical analysis of the children as shown in Table 2.

Table 2. Children Vs Symptoms

R2	Temperature	Headache	Throat Pain	Muscle Ache	Spot
C1	(0.2,0.4,0.4) (0.6,0.2,0.2) (0.3,0.5,0.2)	(0.2,0.6,0.2) (0.4,0.3,0.3) (0.5,0.2,0.3)	(0.8,0.1,0.1) (0.7,0.1,0.2) (0.9,0.1,0)	(0.5,0.2,0.3) (0.6,0.3,0.1) (0.7,0.2,0.1)	(0.2,0.6,0.2) (0,0.7,0.3) (0.1,0.8,0.1)
C2	(0.8,0.1,0.1) (0.9,0.1,0) (0.6,0.2,0.2)	(0.7,0.2,0.1) (0.6,0.1,0.3) (0.5,0.2,0.3)	(0.1,0.7,0.2) (0.2,0.7,0.1) (0,0.6,0.4)	(0.6,0.2,0.2) (0.8,0.1,0.1) (0.7,0.3,0)	(0.7,0.1,0.2) (0.8,0.1,0.1) (0.6,0.1,0.3)
C3	(0.7,0.2,0.1) (0.6,0.3,0.1) (0.8,0.1,0.1)	(0.6,0.2,0.2) (0.4,0.4,0.2) (0.5,0.2,0.3)	(0.3,0.6,0.1) (0.4,0.3,0.3) (0.5,0.2,0.3)	(0.5,0.1,0.4) (0.4,0.1,0.5) (0.6,0.1,0.3)	(0.2,0.8,0) (0.4,0.5,0.1) (0.1,0.7,0.2)

Table 3. Proposed distance measure between Children and Diseases

R3	Viral fever	Throat problem	Chickenpox	Skin problem	Mumps
$C1$	0.1823	**0.1512**	0.3002	0.2152	0.2767
$C2$	0.2802	0.2796	0.1977	**0.1232**	0.2658
$C3$	0.2230	**0.2101**	0.2813	0.2124	0.2974

From the above table, C_1 is diagnosed with Throat problem, C_2 is diagnosed with Chickenpox and C_3 is diagnosed with Throat problem.

Note: If the distance between a patient and a particular disease is the shortest, the patient is likely to have that disease.

Remark: For more accuracy, we have developed a *C#* computer programming algorithm to compute the values in Table 3.

5. The Algorithm and its *C#* Programme.

Algorithm

Step 1 – Start.

Step 2 – Get number of elements in the intuitionistic fussy set to 'n'.

Step 3 – Get the multiplicity to 'm'.

Step 4 – Do steps 5 to 9 for 'η' times and get each Tifs to TifsArray.

Step 5 – Get 'n' elements to fuzzy set M to matrix setM

a. Set i = 0
b. Get values of $\mu M(xi)$ and $vM(xi)$ to setM [i, 0] and setM [i, 1]
c. Set value of $\pi M(xi)$ to setM [i, 2] as 1 - setM [i, 0] - setM [i, 1]
d. Increment i and repeat steps 5.a to 5.d if $i < n$

Step 6 – Get 'n' elements to fussy set N to matrix setN

a. Set i = 0
b. Get values of $\mu N(xi)$ and $vN(xi)$ to setN [i, 0] and setN [i, 1]
c. Set value of $\pi N(xi)$ to setN [i, 2] as 1 – setN [i, 0] – setN [i, 1]
d. Increment i and repeat steps 6.a to 6.d *if* $i < n$

Step 7 – Find absolute difference of corresponding values in set M and N of fuzzy set to matrix. setAbsoluteDiff, Set Tan^{-1} value of sum of each absolute differences to array setAtanOfAbsoluteDiff

a. Set i = 0
b. Set absolute difference of $\mu M(xi)$ and $\mu N(xi)$ to setAbsoluteDiff[i, 0]
c. Set absolute difference of $vM(xi)$ and $vN(xi)$ to setAbsoluteDiff[i, 0]
d. Set absolute difference of $\pi M(xi)$ and $\pi N(xi)$ to setAbsoluteDiff[i, 0]
e. Set Tan^{-1} value of sum of each absolute differences to setAtanOfAbsoluteDiff[i].
f. Increment i and repeat steps 7.a to 7.e if $i < n$

Step 7 – Calculate value of Tangent Inverse Distance to Tifs as $\frac{1}{2(n+1)}$ Sum of elements in array setAtanOfAbsoluteDiff

Step 8 – Calculate value of Difs as $\frac{1}{\eta}\sum_{i=1}^{n} \times Tifs$.

Step-9 Stop

C♯ Program

```
using System; using System.Linq;
namespace FuzzyMath
{internal class ProgramDifs
{
static int n; static int m;
static double[,] setM; static double[,] setN;
static double[,] setAbsoluteDiff; static
double[] setAtanOfAbsoluteDiff; static
double Tifs; static void Main(string[] args)
{
Console.Clear();
Console.WriteLine("Calculating Multi
Tangent
Inverse Distance (Difs)\n------------------");
Console.WriteLine("\n Enter number of
elements in the intuitionistic fuzzy multi
set (n):");
n = Convert.ToInt32(Console.ReadLine());
Console.WriteLine("\nEnter value for
multiplicity (m):");
m =
Convert.ToInt32(Console.ReadLine());
```

```
double[] TifsArray = new double[m];
double[] ThArray = new double[m]; for
(int j = 0; j < m; j++)
{
ProcessTangentCalc(j);
TifsArray[j] = Tifs;
}double Difs = Math. Round(1 / (double)m
* TifsArray. Sum(), 4);

Console.WriteLine($"\nDifs = {Difs}");
        Console.ReadLine();} static void
ProcessTangentCalc (int j)
{
setM = new double [n, 3]; setN = new
double
[n, 3]; setAbsoluteDiff = new double [n,
3]; setAtanOfAbsoluteDiff = new
double[n]; setMaxOfAbsoluteDiff = new
double[n];
Console.WriteLine("\nNow Collecting
values for set \'M\' - Remember to
separate values with <space>\n");
for (int i = 0; i < n; i++)
{
Console.WriteLine($"Enter MueM{j + 1}
(x{i + 1}) and VueM{j + 1}(x{i + 1}):");

string[] valuesOfM = Console.ReadLine().
Split (' '); setM[i, 0] =
Convert.ToDouble(valuesOfM[0]); setM[i,
1] = Convert.ToDouble(valuesOfM[1]);
setM[i, 2] = Math.Round(1 - setM[i, 0] -
setM[i, 1], 2);
}
Console.WriteLine("\nNow, Collecting
values for set \'N\' - Remember
to separate values with <space>\n");
for (int i = 0; i < n; i++)
{
Console.WriteLine($"Enter MueN{j + 1}
(x{i + 1}) and VueN{j + 1}(x{i + 1}):");
string[] input = Console.ReadLine().Split('
'); setN[i, 0] =
Convert.ToDouble(input[0]); setN[i, 1] =
Convert.ToDouble(input[1]); setN[i, 2] =
Math.Round(1 - setN[i, 0] -
setN[i, 1], 2);
}

for (int i = 0; i < n; i++)
{
        setAbsoluteDiff [i, 0] = Math.
Abs (setM [i, 0] - setN[i, 0]);
        setAbsoluteDiff[i, 1] =
Math.Abs(setM[i, 1] - setN[i, 1]);
        setAbsoluteDiff[i, 2] =
Math.Abs(setM[i, 2] - setN[i, 2]);
setAtanOfAbsoluteDiff[i] = Math.Atan(
setAbsoluteDiff[i, 0] + setAbsoluteDiff[i,
1]
+ setAbsoluteDiff[i, 2]);
Tifs = Math.Round(1 / (2 * ((double)n +
1)) *
setAtanOfAbsoluteDiff.Sum(), 4);
}
}
}
```

6. Conclusion

We have developed a new distance measure for intuitionistic fuzzy multisets that extends the Tangent Inverse distance we already defined for IFSs. This method's prominent feature is that it takes into account multi membership, non-membership, and hesitation functions. We have applied the proposed distance measure in the field of medical diagnosis to demonstrate its effectiveness. Also, we have developed a *C♯* computer programming algorithm to compute distance for Intuitionistic Fuzzy Multisets.

References

1. *Krassimir T Atanassov. Intuitionistic fuzzy sets. In Intuitionistic fuzzy sets, pages 1–137. Springer, 1999.*

2. *J Dhivya and B Sridevi. A new similarity measure between intuitionistic fuzzy multisets based on their cardinality with applications to pattern recognition and medical diagnosis. Asian Journal of Research in Social Sciences and Humanities, 7(3):764–782, 2017.*

3. *PA Ejegwa. New operations on intuitionistic fuzzy multisets. J. of Mathematics and Informatics, 3:17–23, 2015.*

4. *PA Ejegwa. Some operations on intuitionistic fuzzy multisets. Journal of Fuzzy Mathematics, 24(4):761–768, 2016.*

5. *Jeswin George and Shiny Jose. Medical diagnosis in intuitionistic fuzzy context. Notes on Intuitionistic Fuzzy Sets, 25:59–63, 04 2019.*

6. *Jeswin B. George and Shiny Jose. Tangent similarity measure of similarity measure based on hausdorff distance. International Journal of emerging technologies and innovative research, 6(6):205–210, 2019.*

7. *P Rajeswari and N Uma. Correlation measure for intuitionistic fuzzy multi sets. International Journal of Research in Engineering and Technology, 3(1):611–617, 2014.*

8. *P Rajeswari and N Uma. Zhang and Fu's similarity measure on intuitionistic fuzzy multi sets. International Journal of Innovative Research in Science, Engineering and Technology, 3(5):12309– 12317, 2014.*

9. *TK Shinoj and Sunil Jacob John. Intuitionistic fuzzy multisets and its application in medical diagnosis. World Academy of Science, Engineering and Technology, 6(1):1418–1421, 2012.*

10. *TK Shinoj and John J Sunil. Accuracy in collaborative robotics: an intuitionistic fuzzy multiset approach. Global J. of Sc. Frontier Research Math. and Decision Sciences, 13(10):21–28, 2013.*

11. *Eulalia Szmidt. Similarity measures between intuitionistic fuzzy sets. In Distances and similarities in intuitionistic fuzzy sets, pages 87–129. Springer, 2014.*

12. *Ronald R Yager. On the theory of bags. International Journal of General System, 13(1):23–37, 1986.*