BURNING NUMBER FOR A FEW GRAPHS USING THE LINE GRAPH MEDIUM

ABSTRACT

In this paper, we are working on Line graphs in order to find the burning number for Path graph, Cycle graph, Star graph, Tadpole graph, Candy graph & Wheel graph. The burning number of a graph G denoted by b(G) is the minimum number of burning steps that is required to completely burn a graph. The burning number varies based on the choice of a source of the vertex at each step and it is always the smallest number that is required in the entire process of burning. Burning a line graph is a simple process to find a burning number.

Keywords: *Line Graph, Burning number.*

1. Introduction:

In this paper, we are considering graph G as a simple graph. Graph G is an ordered triple G = (V(G), E(G) and I(G)) where V(G) is a nonempty set, E(G) is a set disjoint from V(G),I(G) is an " incidence'' relation that associates with each of the elements of E(G) and unordered pair of elements (same or distinct) of V(G). Elements of V(G) are called vertices of G, and elements of E(G) are called the edges of G. V(G) & E(G) are the vertex set and edge set of G where $I_G(e)$ = uv.[2]

The burning process involves discrete steps; each vertex is either burnt or unburnt. To burn a graph at least we need one line. Usually the burning process begins with any vertex. In the first round choose any vertex to burn, but, in the second round if you choose any unburnt vertex then only the first round chosen vertex adjacent will get burnt. Repeat the process until all the vertices are burnt. The chosen vertices are called the source of the vertices. **Once the chosen vertices are burnt, its adjacent vertices will get burnt only by selecting the unburnt vertices.** The burning number of a graph G, denoted by b(G), is the minimum number of rounds needed for the process to end.[4]

Line graph is an undirected simple graph of G, represented as L(G). Each vertex of L(G)denotes an edge of G. The name line graph comes from a paper by Harary & Norma.[3]

Thus when G is the burning graph denoted as b(G) as well as a line graph L(G). The burning number of L(G) is denoted as bL(G).

Burning a number is an information diffusion process to spread information widely in social networks like face book,

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twitter….etc., starts with one burnt vertex called the information sources. It is proposed to model the spread of social contagion, like the communication of disease from one person to another. 'Burning a Graphs as a model of social contagion' by Antony Bonato et al., in the year 2014. **Burning a number** concept was introduced by Antony Bonato.[4]

2. Definitions:

2.1. Line graph: Let G be a simple graph. It is called the line graph or the edge graph of G.[2] When the vertex set of L(G) is in 1-1 correspondence with the line set of G and two vertices of L(G) are joined by line if and only if the corresponding lines of G are adjacent in G.

Simple properties of line graph L(G)

- G is connected if and only if L(G) is connected
- If H is a sub graph of G, then L(H) is also a sub graph of L(G)
- The lines incident at vertices of G give rise to a maximal complete sub graph of L(G).

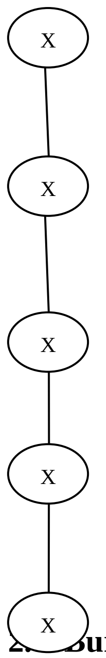
2.2. Burning number: The burning number of a graph G is the minimum number of burning steps required to burn the graph, it is denoted by b(G).[1]

2.3. Star Graph: A Star is a tree as well as Complete bipartite K(1,n) graph and it is denoted by S_k

2.4. Tadpole: A cycle graph C_n, where $n \geq 3$ with an extension of a bridge to a path on any vertex of a cycle is called a Tadpole graph. The path which is connected to on a cycle is called bridge. Tadpole graph is a special type of (m,n)graph. The m refers to the vertices of a cycle and the n refers to the edges of a path. $T_{m,n}$ is the notation of a Tadpole graph.

2.5. Candy graph: A complete graph with m vertices and a path graph on *n* vertices connected with a bridge is called Candy graph. The (m,n) is a candy graph, where m is a complete graph K_m , with the minimum number of the three vertices to a path graph P_n with a bridge. $cd_{m,n}$ is the notation of Candy graph.

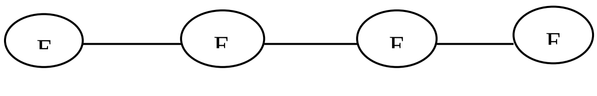
2.6. Wheel graph: A wheel graph has a cycle structure. The inner single center vertex connects to all the vertices of the cycle is called spokes, It is denoted by Wn. For any wheel graph, minimum numbers of 4 vertices are required.



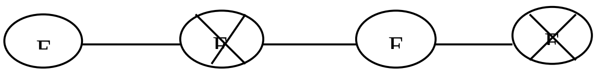
2. Burning number for a L(P) graph

Crossed vertices are burnt and uncrossed vertices are either source vertices or burning number of a L(P) graph. Hence the burning number of L(P_5) is 2.

P_5



L(P5)



bL(P5)

2.7.1. Theorem: A line of a path L(P_n) graph is burning, only if $n \geq 3$.

Proof: The line graph L(G) is derived from a simple connected graph of G. In the line graph L(G) the vertices would be considered as a edges of G. To burn a graph G, minimum two vertices are required. Clearly it shows that to burn a line graph, minimum three vertices are required.

The burning number of L(P_5) is 2.The burning number of a L(P_n) graph is exactly$\sqrt{n-1}$, if and only if the square root of n-1 is a perfect square, otherwise it is $\sqrt{n-1}$ +1.

Case(i): The burning number of L(P_n) graph is exactly$\sqrt{n-1}$, if and only if the square root of n-1 is a perfect square.

Proof: The above case (i), is trivially true where the square root of n-1 is a perfect square and it must be n≥ 5.

Let us discuss how to find the burning number of L(P5)

Burning number for L(Pn) = $\sqrt{n-1}$

Burning number for L(P5) = $\sqrt{5-1}$, is 2.

Case(ii):If n-1is not a perfect square, then the burning number for L(P_n) graph is exactly $\sqrt{n-1}$+1.

Proof: The above case (ii) is true when, n $\geq$ 6 but we need a little calculation. For example let us find the burning for L(P_{20})

Burning number for L(Pn) = $\sqrt{n-1}$+ 1

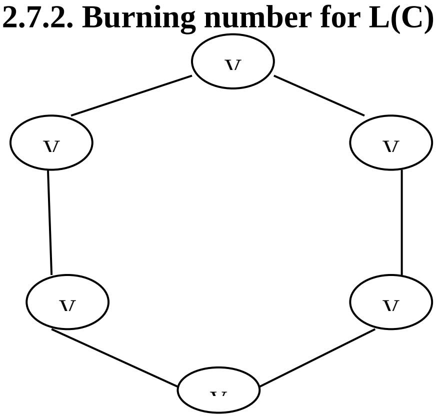
Burning number for L(P_{20}) = $\sqrt{20-1}$ + 1, $\sqrt{19}$+ 1

Therefore burning number of L(P_{20}) = 5

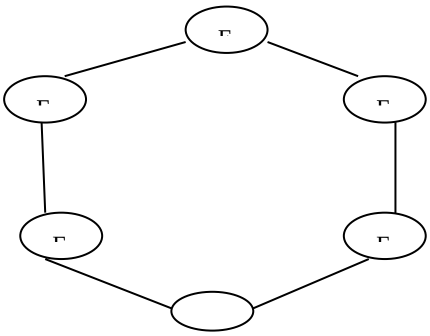
(Consider a perfect square closer to 19; it must be less than the square root of 19. Number 16 is a perfect square which is closer to 19)

Note: The burning number for L(P_3) =L(P_4) = 2.

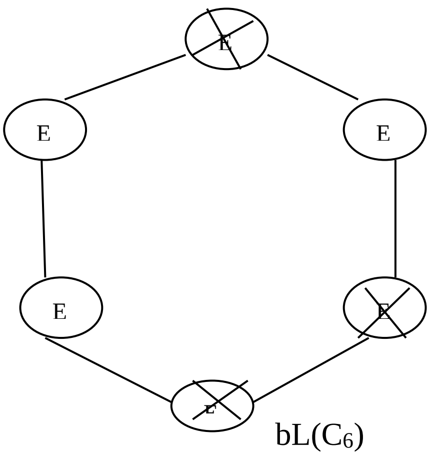
2.7.2. Burning number for L(C) graph



C6



L(C6)



bL(C_6)

Crossed vertices are burnt and uncrossed vertices are either source vertices or burning number of L(C) graph. The burning number for L(C_6) graph is 3.

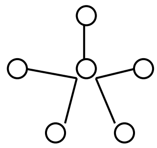
2.7.3. Theorem: For a line of a Cycle graph L(C_n)is burning, only if $n \geq 3$

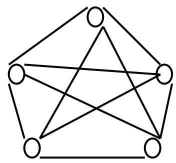
Proof: The burning number for L(C_n) graph is exactly$\sqrt{n}$, when n is a perfect square otherwise it is $\sqrt{n}+1$.When n is a perfect square,it is easy to find it. Suppose if n is not a perfect square, we need a little calculation further.

If n is not a perfect square, then consider the number which a perfect square closer is to the square root of n, but it should not be greater than $\sqrt{n}$ and then add it to 1.

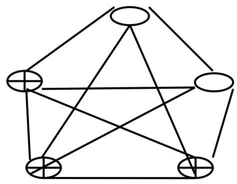
Note: In C_n, the above procedure is same for both even and an odd integer.

2.7.4. Burning number for L($S_{1,n}$) graph





S_5 L(S_5)



bL(S_5)

Crossed vertices are burnt and uncrossed vertices are source vertices or burning number for L**(S_5)** graph is two. The burning number for L($S_{1,n}$) graph is 2.

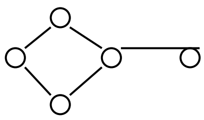
2.7.5. Theorem: Prove that a line of a Star graph, the burning number L($S_{1,n}$) = 2,where n is greater than or equal to three.

Proof: In a line graph, a Star $S_{1,n}$ graph is converted into a complete k_n graph. In a complete graph, all vertices are adjacent to each other.

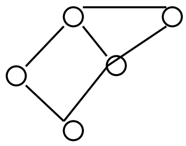
When $n \geq 3$, the burning number of a complete graph b(k_n)is 2 and the burning number of a star graph $S_{1,n}$ is always2.

Therefore, the burning number of a $L(S_{1,n})$ graph is 2 only when n is greater than or equal 3.

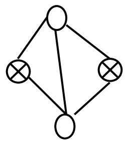
2.7.6. Burning number of $L(t_{m,n})$



$t_{4,1}$



$L(t_{4,1})$



$bL(cd_{3,1})$

Crossed vertices are burnt and uncrossed vertices are source vertices or burning number of $L(T_{4,1})$ tadpole graph is 2.

2.7.7. Theorem: The $t_{m,n}$ graph is burning, the burning number of $L(t_{m,n})$ is $\sqrt{m+n}$, if m+n is a perfect square, otherwise it is $\sqrt{m+n}$ +1.

Proof: The m[th] cycle, connected to P_n path. It is denoted by $t_{m,n,}$ where m is a cycle and n is a path. In a line graph G, the tadpole $t_{m,n}$ graph, transforms to C_m cycle. In C_m cycle, keeping a base on any two vertices forms a triangle and it is connected to P_n path.

We know that $T_{3,n}$is a candy graph structure. The burning number of $L(cd_{3,n})$ is$\sqrt{n+3}$, if n+3 is a perfect square, otherwise $\sqrt{n+3}$ + 1.

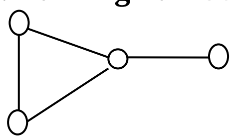
Similarly the burning number of a $L(t_{m,n})$ tadpole graph is $\sqrt{m+n}$ if m+n is a perfect square, otherwise it is $\sqrt{m+n}$ +1.

If m+n is a perfect square, it is easy to calculate the burning number of a $L(t_{m,n})$ graph, if m+n is not a perfect square then we need further calculation process.

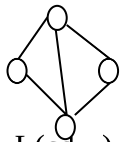
Consider a perfect square number which is closer to the square root of m+n and not exceed the square root of m+n and then add it to 1.

Note: The above result is true when $m \leq n$ for $t_{m,n}$ graph, almost holds for the remaining graphs too.

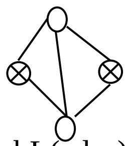
2.7.8. Burning number of L(cd) graph



$cd_{3,1}$



$L(cd_{3,1})$



$bL(cd_{3,1})$

The burning number of $L(cd_{3,1})$ graph is 2 but not for all the burning number of $L(cd_{n,1})$ graphs, a burning number depends on the n[th] integer of $L(cd_{3,n})$.

2.7.9. Theorem: Burning number for a $L(cd_{3,n})$ is$\sqrt{n+3}$, if n+3 is a perfect square otherwise $\sqrt{n+3}$ + 1.

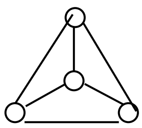
Proof: A Candy graph $cd_{3,n}$ is a complete graph with the k_3 vertices connected to P_n path. A line of a candy graph $L(cd_{3,n})$ is derived from candy $cd_{3,n}$ graph. One of the diagonal which is joined to the inside of a square and it is connected to n-1 path on any base of the square, is called a line of a candy $L(cd_{3,n})$ graph.

The total number of vertices in a $L(cd_{3,n})$ is 3+n.

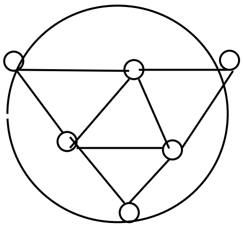
The burning number for a $L(cd_{3,n})$ candy graph is $\sqrt{n+3}$ only if n+3 is a perfect square, otherwise it is $\sqrt{n+3}$+ 1. For this, we need a further calculation. Suppose, if it is not a perfect square while adding n+3, then take a perfect square number which is closer to the square root of n+3which should not exceed the square root of n+3, and then add it to one.

Note: $L(cd_{3,}1) = L(cd_{3,}2) = 2$.

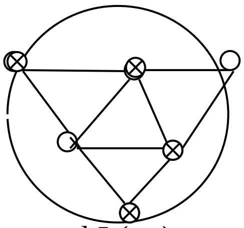
2.7.10. Burning number of a line of a wheel graph:



w_4



$L(w_4)$



$bL(w_4)$

Crossed vertices are burnt and uncrossed vertices are either source vertices or burning number.

The burning number for $L(w_4)$ is 2.

2.7.11. Theorem: The burning number for a $L(w_n)$ is 2, where n = 4, otherwise it is 3

Proof: According to the w_n graph definition, where $n \geq 4$ is a wheel graph. $L(w_n)$ is a transformation of w_n. The burning number for w_n graph is 2.

$L(w_n)$ is a k_{n-1} complete graph inscribed in a circle, where n-1 triangular forms on the top of a complete graph vertices. In a complete graph, where all the vertices are adjacent to each other, the complete graph k_n burning number is 2. Therefore, the burning number for $L(w_n)$ is 2, when n=4,otherwise it is 3.

Note: Burning number for $L(w_4) = 2$,

Burning number of $L(w_n) = 3$ when $n \geq 5$.

3. Conclusion: In this paper, we have successfully generalized the burning number for line graph such as Path graph, Cycle graph, Star graph, Tadpole graph, Candy graph & Wheel graph.

References

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