SOME RESULTS ON HARMONIC MEAN GRAPHS

ABSTRACT

A graph G with p vertices and q edges is called a harmonic mean graph if it is possible to label the vertices x∈V with distinct labels f(x) from {1,2,.....q+1} in such a way that each edge e = uv is labeled with f(uv) = $\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil$ *(or)* $\left\lfloor\frac{2f(u)f(v)}{f(u)+f(v)}\right\rfloor$ *then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove that some families of graphs such as Tad pole T (n,t) , T (n,t)* $\odot K_1$,
T (n,t) $\odot \overline{K_2}$,*T (n,t)* $\odot K_2$ *are harmonic mean graphs.*

Keywords: *Harmonic mean graph, Tad pole T (n,t) , T (n,t)* $\odot K_1$, *T (n,t)* $\odot \overline{K_2}$, *T (n,t)* $\odot K_2$.

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1. Introduction:

Let G=(V,E) be a (p,q) graph with p = |V(G)| vertices and q =|E(G)| edges, where V(G) and E(G) respectively denote the vertex set and edge set of the graph G. In this paper, we consider the graphs which are simple, finite and undirected. For graph theoretic terminology and notations we refer to Harary [4]

The concept of graph labeling was introduced by Rosa [1] in 1967. A detailed survey of graph labeling is available in Gallian[6].The concept of Harmonic mean labeling of graph was introduced by S.Somasundaram, R.Ponraj and S.S.Sandhya and they investigated the existence of harmonic mean labeling of several family of graphs such as path, comb, cycle C_n, in [10,11]. The following definitions are useful for the present investigation.

Definition: 1.1

A Graph G = (V, E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices x ∈V with distinct labels f(x) from {1,2,…,q+1} in such a way that when each edge e = uv is labeled with f(uv) = $\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil$ (or)

$\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition: 1.2

A tadpole T (n,t) is the graph obtained by appending a path P_t to a cycle C_n .

In this paper we prove that Tad pole T (n,t), T (n,t) $\odot K_1$, T (n,t) $\odot \overline{K_2}$, T (n,t) $\odot K_2$ are harmonic mean graphs.

2. Harmonic mean labeling of graphs

Theorem:2.1

The tadpole T (n,t) is a harmonic mean graph.

Proof:

Let $u_1u_2, \ldots, u_n u_1$ be the cycle C_n and let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n which are joined to the vertex u_i of cycle C_n, $1 \leq i \leq n$. Then the resultant graph is tadpole T (n,t).

Define a function f :V(G) $\rightarrow \{1,2, \ldots, q+1\}$ by

$f(u_i)$ = i for $1 \leq i \leq n$

$f(v_i)$ = n+i for $1 \leq i \leq n$

Then the resulting edge labels are distinct.

$f(u_n u_1)$ = 1

$f(u_{i-1} u_i)$ = n for $2 \leq i \leq n-1$

$f(u_n v_1)$ = n+1

$f(v_i v_{i+1})$ = (n+1) + i

Thus, f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph.

Example:2.2

A harmonic mean labeling of tadpole T (5,6) is given in fig 2.2

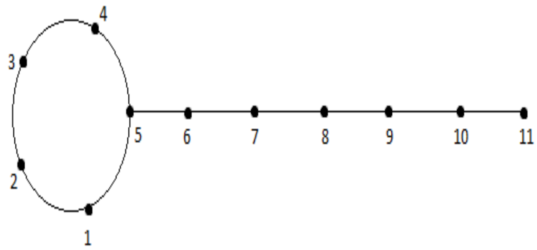

Fig (2.2)

Theorem:2.3

The tadpole T (n,t) $\odot K_1$ is a harmonic mean graph.

Proof:

Let G be the tadpole graph, and let $v_1, v_2, \ldots, v_n$ be the vertices of K_1 which are joined to the vertex u_i of the cycle C_n , $1 \leq i \leq n$. Let $x_1, x_2, \ldots, x_n$ be the vertices of K_1 which are joined to the vertex w_i of the path P_n , $1 \leq i \leq n$. Then the resultant graph is T (n,t) $\odot K_1$ graph.

Define a function f:V(G)$\rightarrow \{1,2, \ldots, q+1\}$ by

$f(u_i)$ = 2i for $1 \leq i \leq n$

$f(v_i)$ = 2i -1 for $1\leq i \leq n$

$f(w_i)$ = 2n+2i for $1\leq i \leq n$

$f(x_i)$ = (2n+1)+ 2(i-1) for $1\leq i \leq n$

Then the resulting edge labels are distinct.

$f(u_1v_1) = 1$

$f(u_iv_i)$ = 2i for $2\leq i \leq n$

$f(u_nu_1)$ = 3

$f(u_1u_2)$ = 2

$f(u_iu_{i+1})$ = 2i+1 for $2\leq i \leq n-1$

$f(u_nw_1)$ = 2n+1

$f(w_iw_{i+1})$ =2n+ 3+ 2(i-1) for $2\leq i \leq n-1$

$f(w_ix_i)$ = 2n+ 2+ 2(i-1) for $2\leq i \leq n$

Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph.

Example:2.4

A harmonic mean labeling of tadpole T (7,7) ⊙ K_1 is given in fig 2.4

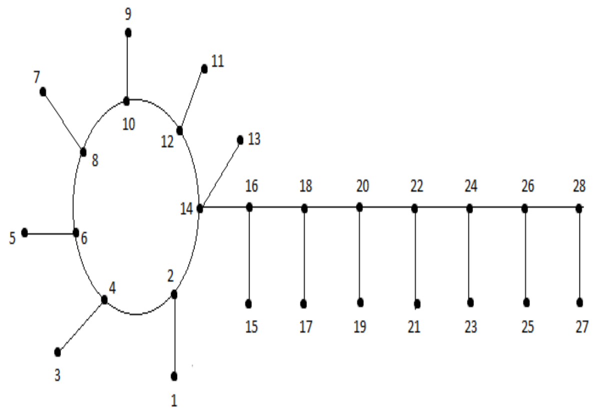

Fig (2.4)

Theorem:2.5

The tadpole T (n,t) ⊙ $\overline{K_2}$ is a harmonic mean graph.

Proof:

Let G be the tadpole graph, and let w_i, v_i be the vertices of $\overline{K_2}$ which are joined to the vertex u_i of the cycle C_n , $1\leq i \leq n$. Let y_i, z_i be the vertices of $\overline{K_2}$ which are joined to the vertex x_i of the path P_n , $1\leq i \leq n$. Then the resultant graph is T (n,t) ⊙ $\overline{K_2}$ graph.

Define a function f : V(G) → $\{1,2,\ldots,q+1\}$ by

$f(u_i)$ = 3i -1 for $1\leq i \leq n$

$f(v_i)$ = 3i -2 for $1\leq i \leq n$

$f(w_i)$ = 3i for $1\leq i \leq n$

$f(x_i)$ = 3n+2+ 3(i-1) for $1\leq i \leq n$

$f(y_i)$ = 3n+1+ 3(i-1) for $1\leq i \leq n$

$f(z_i)$ = 3n+3+ 3(i-1) for $1\leq i \leq n$

Then the resulting edge labels are distinct.

$f(u_1u_2)$ = 3

$f(u_iu_{i+1})$ = 3i+1 for $2\leq i \leq n-1$

$f(u_1v_1)$ = 1

$f(u_iv_i)$ = 3i-1 for $2\leq i \leq n$

$f(u_1w_1)$ = 2

$f(u_iw_i)$ = 3i for $2\leq i \leq n$

$f(u_nx_1)$ = 3n+1

$f(x_ix_{i+1})$ = 3n+ 1+ 3i for $1\leq i \leq n-1$

$f(x_iy_i)$ = 3n+ 2+ 3(i-1) for $1\leq i \leq n$

$f(x_i z_i)$ = 3n+ 3+ 3(i-1) for $1 \leq i \leq n$

Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph.

Example:2.6

A harmonic mean labeling of tadpole T (7,5) $\odot$ $\overline{K_2}$ is given in fig 2.6

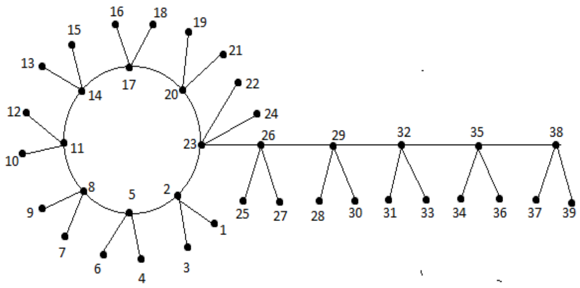

Fig (2.6)

Theorem:2.7

The tadpole T (n,t) $\odot$ K_2 a harmonic mean graph.

Proof:

Let G be the tadpole graph, and let w_i, v_i be the vertices of K_2 which are joined to the vertex u_i of the cycle C_n, $1 \leq i \leq n$. Let y_i, z_i be the vertices of K_2 which are joined to the vertex x_i of the path P_n, $1 \leq i \leq n$. Then the resultant graph is T (n,t) $\odot$ K_2 graph.

Define a function f : V(G)→ $\{1,2,\dots,q+1\}$ by

$f(u_i)$ = 4i -1 for $1 \leq i \leq n$

$f(v_1)$ = 1

$f(v_i)$ = 4i for $2 \leq i \leq n$

$f(w_1)$ = 4

$f(w_i)$ = 4i -3 for $2 \leq i \leq n$

$f(x_1)$ = 4n+3

$f(x_{i+1})$ = 4n+3+ 4i for $1 \leq i \leq n$ -1

$f(y_1)$ = 4n+1

$f(y_{i+1})$ = 4n+1+4i for $1 \leq i \leq n-1$

$f(z_1)$ = 4n+4

$f(z_{i+1})$ = 4n+4+4i for $1 \leq i \leq n-1$

Then the resulting edge labels are distinct.

$f(u_1 u_2)$ = 4

$f(u_i u_{i+1})$ = 4i+1 for $2 \leq i \leq n-1$

$f(u_n v_1)$ = 5

$f(u_1 v_1)$ = 1

$f(u_i v_i)$ = 4i for $2 \leq i \leq n$

$f(u_1 w_1)$ = 3

$f(u_i w_i)$ = 4i -2 for $2 \leq i \leq n$

$f(v_1 w_1)$ = 2

$f(v_i w_i)$ = 4i - 1 for $2 \leq i \leq n$

$f(u_n x_1)$ = 4n+1

$f(x_i x_{i+1})$ = 4n+ 1+ 4i for $1 \leq i \leq n-1$

$f(x_1 y_1)$ = 4n+2

$f(x_{i+1} y_{i+1})$ = 4n+ 2+ 4i for $1 \leq i \leq n-1$

$f(x_1 z_1)$ = 4n+4

$f(x_{i+1} z_{i+1})$ = 4n+ 4+ 4i for $1 \leq i \leq n-1$

$f(y_1 z_1)$ = 4n+3

$f(y_{i+1} z_{i+1})$ = 4n+ 3+ 4i for $1 \leq i \leq n-1$

Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph.

Example:2.8

A harmonic mean labeling of tadpole T (8,5) ⊙ K_2 is given in fig 2.8

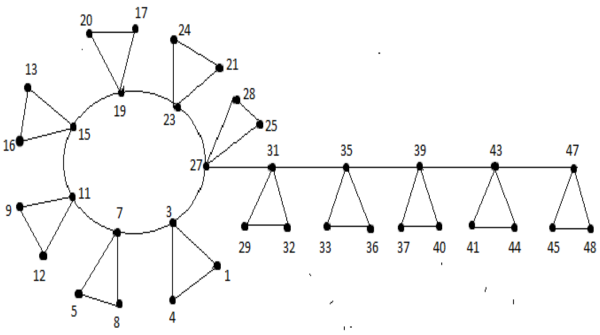

Fig (2.8)

Conclusion:

We have presented new results on Harmonic mean labeling of certain classes of graphs like the Tad pole T (n,t) , T (n,t) ⊙ K_1, T (n,t) ⊙ $\overline{K_2}$ .Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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