MEAN NUMBER OF SOME GRAPHS

ABSTRACT

A graph G consists of a finite set of vertices V(G) and set of edges E(G) there is no common label in this two sets. labeling on a graph is the assignment of an integer value to the elements of the graph. We have laid out a general organization of mean number example of P_r; $P_s \odot K_1; T(P_n); Hd_t$. ……

Keywords: *Mean number, mean graph, total path graph.*

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Introduction:

"Somasundaram and Ponraj have introduced the mean labeling" [5]. In this paper graph G refers to simple graph definition and notations in graph theory follow "Bondy Murthy" [2] throughout this paper we discuss Mean number of certain graphs here MNL denotes Mean number labeling, ML-Mean labeling.

Definition 1.1

The comb graph is defined by $P_n \odot K_1$. It has $2n$ vertices and $2n-1$ edges.

Definition 1.2

Let G be a (p,q) graph. A function f is called Mean Graph of G. It is possible to label the vertices $v \in V$ with distinct label $f(x)$ from $1,2,3, \ldots., s$ in such a way that when each e is labeled with $f(e) = \left[\frac{f(x)+f(y)}{2}\right]$ $if\ f(x) + f(y)\ is\ even\ and$ $\left[\frac{f(x)+f(y)+1}{2}\right]$ $if\ f(x)+f(y)\ is\ odd$ then the resulting edge labels are distinct. In this case f is called mean graph.

Definition 1.3

G be a graph and g:V$(G) \to \{1,2,3, \ldots, p+q\}$ is a function such that the labels of the edge uv is $g(e) = \left\{ \begin{matrix} \left[\frac{g(u)+g(v)}{2}\right] \\ \left(\left[\frac{g(u)+g(v)+1}{2}\right]^{or}\right) \end{matrix} \right\}$ and $\{(g(V(G)) \cup (g(e)) \mid e \in E(G)\} \subseteq \{1,2,3, \ldots, n\}$. If n is the least positive integer which satisfies there is no common labels in vertex and

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edge is called the mean number of graph G and is denoted by m(G).

2. Important Result

Theorem 2.1

For the path P_r , r ≥ 3 is a mean graph, Then $m(P_r) = 2r - 1$.

Proof:

Let $G = P_r$, with $n(G) = r$ and $e(G) = (r - 1)$.

$$V(G) = \{v_j\}_{j\in[1,r]}$$

$$E(G) = \{e_j; 1 \le j < r\}$$

$$N(v_j) = \{v_{j-1}, v_{j+1};\}_{j\in(1,r)}$$

$$N(v_1) = \{v_2\}\ and$$

$$N(v_r) = v_{r-1}$$

$$ML(v_j) = \{(2j-1): j\ varies\ from\ 1\ to\ r\}$$

$$ML(e_j) = \{2j; 1 \le j \le (r-1)\}$$

There are no common labels in vertex and edge.

Thus, G satisfies the mean number condition.

Therefore, $m(P_r) = 2r - 1$.

Theorem 2.2

For $n \ge 3, P_s \odot K_1$ is a mean graph then $m(P_s \odot K_1) = 4s - 1$.

Proof:

Let $G' = P_s \odot K_1$, $|V(G')| = 2s\ and\ |E(G')| = 2s - 1$.

$$V(G') = \{v_i, v_i'; (1 \le i \le s)\}$$

$$E(G') = \begin{Bmatrix} e_i; 1 \le i < s \\ e_i'; 1 \le i \le s \end{Bmatrix}$$

$$N(v_i) = \{v_{i-1}, v_{i+1}, v_i'; 1 < i < s\}$$

$$N(v_1) = \{v_2, v_1'\}\ and\ N(v_s) = \{v_{s-1}, v_s'\}$$

$$f(e_i) = \{v_i v_{i+1}; 1 \le i < s\}$$

$$f(e_i') = \{v_i v_i'; i\ varies\ from\ 1\ to\ s\}$$

The vertex and edge mean number labeling pattern of G are,

$$ML(v_i) = \begin{Bmatrix} 4i - 1\ if\ i\ is\ even \\ 4i - 3\ if\ i\ is\ odd \end{Bmatrix}$$

$$ML(e_i) = \{4i;\}_{i\in[1,s-1]}$$

$$ML(e_i') = \{4i - 2; i = 1\ \ to\ s\}$$

$$f(V(G') \cap E(G')) = \emptyset$$

Thus we have, $m(P_s \odot K_1) = 4s - 1$

Theorem 2.3

The total path graph $T(P_n)$ is mean graph and its mean number is $2(3n - 3)$.

Proof:

Given, $T(P_n)$ is a mean graph,

$$|V(T(P_n))| = 2n - 1\ and$$

$$|E(T(P_n))| = 4n - 5.$$

$$V(T(P_n)) = \begin{Bmatrix} v_i; 1 \le \mathrm{i} \le \mathrm{n} \\ \mathrm{v}_\mathrm{i}'; 1 \le i < n \end{Bmatrix}$$

$$E(T(P_n)) = \begin{Bmatrix} e_i; 1 \le i < n \\ e_i'; 1 \le \mathrm{i} \le (2\mathrm{n} - 2) \\ \mathrm{e}"_\mathrm{i}; 1 \le i \le (n-2) \end{Bmatrix}$$

$$N(v_i) = \{v_{i+1}, v_{i-1}', v_{i-1}, v_i'; 1 < i < n\}$$

$$N(v_1) = \{v_2, v_1'\}; N(v_n) = \{v_{n-1}, v_{n-1}'\}$$

$$N(v_i') = \{v_{i-1}', v_{i+1}', v_i, v_{i+1}; 1 < \mathrm{i} < (\mathrm{n} - 1)\}$$

$$N(v_1') = \{v_2', v_1, v_2\}; N(v'_{n-1}) = \{v_{n-1}, v_{n-2}, v_n\}$$

$$f(e_i) = \{v_i v_{i+1}\}_{i\in[1,n-1]}$$

$$f(e'_{2i-1}) = \{v_i v'_i; 1 \le i \le n\}$$

$$f(e''_i) = \{v'_i v'_{i+1}; i\ varies\ from\ 1\ to\ (n-2)\}$$

The MNL pattern of G is,

$$ML(v_1) = 1;$$

$$ML(v_i) = \{2(3i-3)\ i\ varies\ from\ 2\ to\ n;\}$$

$$ML(v'_1) = 3;$$

$$ML(v'_i) = \{2(3i-1); i\ varies\ from\ 1\ to\ (n-1)\}$$

$$ML(e_1) = 4;$$

$$ML(e_i) = \{6i-3; i\ varies\ from\ 2\ to\ n\}$$

$$ML(e'_i) = \{3i-1; i\ varies\ from\ 1\ to\ 2n-2\}$$

$$ML(e''_i) = \{6i+1; i\ varies\ from\ 1\ to\ n-1\}$$

$$f(n(T(P_n))) \cap f(e(T(P_n))) = \emptyset$$

Hence, $m(T(P_n)) = 2(3n-3)$

Theorem 2.4

For the Hurdle graph Hd_t is mean graph and its $m(Hd_t) = 4t-5$.

Proof:

Let $G = Hd_t$ here $2t-2$ vertices and $2t-3$ edges.

$$V(G) = \begin{Bmatrix} u_i; i = 1\ to\ t \\ u'_i; i\ varies\ from\ 1\ to\ (t-2) \end{Bmatrix}$$

$$E(G) = \begin{Bmatrix} e_i; i\ varies\ from\ 1\ to\ t \\ e'_i; i = 1,2,\dots,(t-2) \end{Bmatrix}$$

$$N(u_i) = \{u_{i-1}, u_{i+1}, u'_i, u'_{i-1}; i = 2,3,\dots,(t-1)\}$$

$$N(u_1) = \{u_2\}; N(u_t) = \{u_{t-1}\}$$

$$N(u'_i) = \{u_{i+1}\}$$

$$f(e_i) = \{u_i u_{i+1}; i = 1,2,\dots,(t-1)\}$$

$$f(e'_i) = \{u_{i+1} u'_i; i\ varies\ from\ 1\ to\ t-2\}$$

We describe a mean number labeling as follow,

$$ML(u_i) = \begin{Bmatrix} 4i-3\ if\ i\ is\ odd \\ 4i-5\ if\ i\ is\ even \end{Bmatrix}$$

$$ML(u'_i) = \begin{Bmatrix} 4i-1\ if\ i\ is\ odd \\ 4i+1\ if\ i\ is\ even \end{Bmatrix}$$

$$ML(e_i) = \{4i-2\ ; i = 1,2,\dots,t-1\}$$

$$ML(e'_i) = \{4i; i\ varies\ from\ 1\ to\ t-2\}$$

$$f(V(G)) \cap f(E(G)) = \emptyset$$

Hence $m(Hd_t) = 4t-5$.

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