ON GENERALIZED SEQUENTIALLY CONNECTEDNESS

ABSTRACT

We discuss the definition of sequentially-g-connectedness by utilizing sequentially- g-closed sets. Besides, we investigate some properties of sequentially-g- connectedness. Also, we discuss the set's product, which is sequentially-g-connected.

Keywords: *sequentially-g-open, g-converges, sequentially-g-continuous, sequentially-g-connected.*

1. Introduction and Preliminaries:

Levine [7] created the notion of a topological space's generalised closed set (also known as the "g-closed set"). Dunham and Levine [4] thought about the g-closed sets initially, followed by Dunham [3]. They are study of g-closed set that is, If *cl*(L) subset W exists while L subset W and W are open in T, then L subset T is said to be "g-closed" in the topological space (T,τ). let (T,τ) be a topological space and L $\subset$ T is called g-closed if cl(L) $\subset$ W holds whenever L $\subset$ W and W is open in T. Every closed set is g-closed, otherwise, these sets are coincided in $T_{1/2}$ space. In terms of g-open sets, Caldas and Jafari [2] showed a brand-new sort of convergence. They also looked at sequentially g-closed sets and sequential g-continuous mappings.

Balachandran [1] introduced the definition of GO-connected by using *g*-open sets. In this paper we discuss sequentially-g-connected space by using sequentially-g-closed set. L is called *sequentially closed* [5] if for each sequence $(t_n)\in L$ which is converges to *t*, then $t\in L$. L is sequentially openin T if the complement of L is sequentially closed. If L cannot be described even as union of two nonempty separate sequentially open sets of L, then L is referred to as being *sequentially connected* [6]. Let (T,τ) be a topological space. L is a subset of T and T L is denotedby the complement of the set L. Let cl denote the closure operator on $(T,\tau)$and N be the set of all natural numbers. A sequence (t_n) inside a space T *g-converges* to a point $t\in T$ [2] if whenever (t_n) eventually exists throughout each *g*-open set containing *t*;t is represented by (t_n) $\xrightarrow{g} t$, this point is known as the sequence's g-limit and is represented by glimt_n. If a sequence in L *g*-converges to a point in L, then A is said to be *sequentially-g-closed* [2].T [L] denote the

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set of all sequences in L and $c_g(T)$ denote the set of all g- convergent sequences in T. Give any two topological spaces for (T,τ) and (Y, σ). If the sequence $(f(t_n)) \xrightarrow{g} f(t)$ occurs whenever the sequence $(t_n) \to t$ the map $f : (T,\tau) \to (Y,\sigma)$ is called *sequentially-g-continuous* at $t \in T$ [2]. A function is referred to as *sequentially-g-continuous* if it is sequentially-g-continuous at each $t \in T$. Each g-open cover of a space T issaid to be *GO-compact* [1] if it contains a finite subcover

Theorem 1.1. Let (T, τ) be a topological space and $L \subset T$. If L is sequentially closed, then $L = [L]_{seq}$.

Definition 1.2. When a g-open set O contains a $t \in O$ subset L, the subset L of a topological space (T, τ) is referred to as a point's g-neighborhood.

Definition 1.3. [10] Assuming a topological space with the properties (T, τ), $L \subset T$, and $T[L]$, which is the collection of all sequences in L. The sequential g-closure of L, represented by $[L]_{g-seq}$, is therefore given as $[L]_{g-seq} = \{t \in T \mid t = glim\ t_n$ and $(x_n) \in T[L] \cap c_g(T)\}$

All g convergent sequences in T are represented by the set $c_g(T)$. Each g-convergence sequences seems to be a convergence sequences, as shown by the theorem 1.4 (a) that follows.

But converse of Theorem neednot be true by Example 2.5.

Theorem 1.4. [10] Let (T, τ) be a topological space. Then the following hold.

(a) Each g-convergence sequences seems to be a convergence sequences.

(b) Convergence is the same as g-convergence when (T, τ) seems to be a $T_{1/2}$ space.

Example 1.5. Consider the topological space (T, τ) where $T = [0, 2)$, $\tau = \{\emptyset, (0, 1), T\}$. Consider the case when $(t_n) = \frac{1}{n}$ for $n \in N$. As a result, (t_n) converges to 0. In the event that $L = (0, 1]$, $T \setminus L$ is g-open because $L$is g-closed. In other words, T is a g-open subset of $\{0\} \cup (1, 2)$. However, $\frac{1}{n} \notin \{0\} \cup (1, 2)$ for any n. Because of this, (t_n) is not g-convergent to 0.

Lemma 1.6. [10] Suppose that $f:(T,\tau) \to (Y,\sigma)$ is a sequentially g-continuous function. It follows that $f^{-1}(L)$ is sequentially-g-closed whenever L is sequentially-g-closed.

2. Sequentially-g-connectedness

The section covers the characterization of sequentially-g-connectedness.

Theorem 2.1. A sequentially-g-closed set is any set that has been sequentially-g-closed.

Proof. Let L T. Let's say that L is sequentially closed. Then, according to Theorem 1.1, $L = [L]_{seq}$. A sequence in the set L such that $(t_n) \xrightarrow{g} t$ must be defined as (t_n). According to Theorem 1.4 (a), $t \in [L]_{seq}$ is $(t_n) \to t$ in L. Therefore, $t \in L$ As a result, L is sequentially-g-closed.

Lemma 2.2. Let (T, τ) be a topological space. In the event where Y is sequentially-g-closed in T but also L is sequentially-g-closed in Y , thereforeL is sequentially-g-closed in T.

Proof. A sequence g-converging to t in T is defined as $(t_n) \in L$. Since Y is sequentially-g-closed in T and L Y, then t Y. Thus, (t_n) g-converges to t in Y . Moreover, L is sequentially-g-closed in Y, therefore $t \in L$. So, at T, L is sequentially-g-closed.

Definition 2.3. If there exist no nonempty and disjoint sequentially-g-closed subsets O and P such that $L \subseteq O \cup P$, and $L \cap O$, $L \cap P$ are nonempty, then a nonempty subset L of a topological space (T, τ) is said to be sequentially-g-connected [10] Whenever there are no nonempty, disjoint sequentially-g-closed subsets of T whose union equals T, then T is said to be sequentially- g-connected.

Theorem 2.4. [10] Any sequentially-g-connected subset of T is sequentially-g-connected if it has a sequentially-g-continuous image of it.

Theorem 2.5. Assume that $L \subset T$ exists in the topological space (T, τ), after which hold.

(a) In the event that L is a sequentially-g-connected subset of L, then L is sequentially connected in T.

(b) If L is a sequentially connected and sequentially-g-closed in T, then L is sequentially-g-connected in T.

Proof. (a) Let's say that L is not related to T in a sequential manner. There are nonempty sequentially closed subsets of T called O, P that satisfythe condition $L = O \cup P$. According to Theorem 2.1, O and P are also sequentially-g-closed subsets of L. It follows that L is not sequentially-g-connected because O and P are disjoint sequentially-g-closed subsets of L, that is illogical.

(b) Assume that L is sequentially connected in T and sequentially-g-closed in$T$. When $L = O \cup P$, there are nonempty disjoint sequentially closed subsets$O$ and P of L. O and P are sequentially-g-closed subsets of L accordingto Lemma 3.1. By using the lemma 2.2, O and P are nonempty disjoint sequentially-g-closed subsets of T because L is a sequentially-g-closed set within T. Therefore, L is sequentially-g-connected. This is a contradiction.

The following Example 2.6 shows that the condition of sequentially-g-closed sets in Theorem 2.5 (b) is very important.

Example 2.6. Consider the topological space (T, τ) where T = [0, 5), $\tau = \{\phi, (0,1), T\}$. Let L = (0, 1] and B = (2, 5) are subsets of T. Since L and B are nonempty disjoint sequentially closed subsets of T, Then T is a sequentially connected set. Assume that $(s_n) = \frac{1}{n}$ for $n \in$ N. As a result, (s_n) converges to 0. Given that L = (0, 1] is in this case g-closed, $T \setminus L$ is g-open. In other words, T is a g-open subset of $\{0\} \cup (1,5)$. However, $\frac{1}{n} \notin \{0\} \cup (1,5)$ about any n. In light of this, (s_n) doesn't really g-convergeto 0 and so L is not a sequentially-g-closed subset in T. Therefore, T is sequentially-g-connected.

Lemma 2.7. [10] A sequentially-g-connected subset of T is L. In the case where O and P are nonempty disjoint sequentially-g-closed subsets of T and L $\subseteq$ O $\cup$ P, then either L $\subseteq$ O or L $\subseteq$ P.

Lemma 2.8. Let (T, τ) be a topological space and L $\subset$ T, and O a sequentially g-open and sequentially-g-closed subset of T. If L is sequentially-g-connected, then either L $\subseteq$ O or L $\subseteq$ T \O.

Proof. Even if $O = \emptyset$ or $O = T$ *were* assumed, the evidence would still be clear. As a result, using the Lemma 2.7, either $L \subseteq O$ or $L \subseteq T \setminus O$. If $O \neq \emptyset$ and $O \neq T$, therefore $L \subseteq O \cup (T \setminus O)$.

Theorem 2.9. The topological space (T, τ) shall be assumed and L $\subset$ T, and $L \subseteq N \subseteq [L]_{g-seq}$. Assuming that L is sequentially-g-connected, N must likewise be so.

Proof. In the event when $L \subseteq N \subseteq [L]_{g-seq}$, so $N \subseteq [L]_{g-seq} \cap N = [L]_{g|N-seq}$. On either case, L is equal to $[L]_{g|N-seq} \subseteq L$. Since $[L]_{g|N-seq}$ is the sequential *g*-closure of L in L, $[L]_{g|N-seq} = L$. Assume, on the other hand, that N is not sequentially-g-connected. As a result, the sequentially g-closed subsets O and P of T are nonempty and disjoint, and $N \subseteq O \cup P$, $N \cap O$, and $N \cap P$ are nonempty. Given that L is sequentially-g-connected and that$L \subseteq O$ or $L \subseteq P$, respectively. Assuming *N* subsets *O*, so $[L]_{g-seq} \subseteq [O]_{g-seq}$, resulting in $[L]_{g|N-seq} = [O]_{g-seq} \cap L$. The fact that O is sequentially-g-closed in T means that $[O]_{g-seq} = O$. Inferring that $N = N \cap O$ from the fact that$\boldsymbol{L} = [L]_{g|N-seq} = O \cap L$. Similarly, $N = N \cap P$ if $\boldsymbol{L} \subseteq \boldsymbol{P}$. The proof is finished by this inconsistency.

Corollary 2.10. Assuming L is a sequentially-g-connected subset of T and (T, τ) is a topological space, then $[L]_{g-seq}$ too is sequentially-g-connected.

Proof. This proof follows from Theorem 2.9.

Theorem 2.11. A class of sequentially g-connected subsets of T, called $\{M_j : j \in I\}$ shall exists. $\bigcup_{j\in I} M_j$ are sequentially g-connected if $\bigcap_{j\in I} M_j \neq \phi$.

Theorem 2.12. An assortment of sequentially-g-connected subsets of the set T are referred to as $\{A_j : j \in I\}$. Let $L \subseteq T$ sequentially g-connected such that $L \cap A_j$ is nonempty for each $j \in I$. Additionally, $L \cup \left(\bigcup_{i\in I} A_j\right)$ is sequentially-g-connected.

Proof. Since $L \cap A_j \neq \phi$, $M_j = L \cup A_j$ is sequentially-g-connected for each $j \in I$, and $\bigcup_{i\in I} A_j \neq \phi$. Therefore, the union $\bigcup_{i\in I} M_j$ = $L \cup \left(\bigcup_{i\in I} A_j\right)$ is sequentially-g-connected, by Theorem 2.11.

3. The product of sequentially-g-connected spaces

In this section, we discuss about a product property of sequentially *g*-connectedness.

Theorem 3.1. If T and Y are sequentially-g-connected, then $T \times Y$ is sequentially-g-connected.

Proof. Fix $t \in T$. If T is sequentially-g-connected, then $L = \{t\} \times Y$ is sequentially-g-connected as the image of a sequentially-g-connected set under sequentially-g-continuous map $f_x : T \to T \times Y$ defined by $t \to (t, y)$. Similarly, for each $y \in Y$ the subset

$M_y = T \times \{y\}$ is sequentially-g-connected and $L \cap M_y$ has a common point (t, y). Hence $L \cup M_y$ is sequentially-g-connected by Theorem 2.11. Since $T \times Y = \cup_{y \in Y} (L \cup M_y)$. Therefore, $T \times Y$ is sequentially-g-connected by Theorem 2.11.

Theorem 3.2. The countable product space of sequentially-g-connected spaces is sequentially-g-connected.

Proof. Let T and Y be sequentially-g-connected spaces. we take a fixed point $c_0 = (x_0, y_0) \in T \times Y$. Let c = (t, y) be any point of $T \times Y$. It is enough to show that sets $(T \times \{y_0\})$, $(\{x_0\} \times Y)$ are sequentially-g-connectedin $T \times Y$. Next, $(t, y_0) \in (T \times \{y_0\}) \cap (\{x_0\} \times Y)$. Hence $(T \times \{y_0\}) \cup (\{x_0\} \times Y)$ is sequentially-g-connected by Theorem 2.11, which contains points c_0, c. ByTheorem 2.12, $T \times Y$ is sequentially-g-connected. By induction, a finite product space of sequentially-g-connected spaces is sequentially-g-connected. Let $\{X_j\}_{j \in N}$ be a countable family of sequentially-g-connected spaces and let $T = \prod_{j \in N} X_j$ be the countable product space. Fix a point a point $\beta = (\beta_j) \in T$. For each

$n \in N$, put $C_n = \left(\prod_{j \le n} X_j \right)$, then

$C_n \subset C_{n+1}$. So C_n is a sequentially-g-connected subspace of T by the finite case of the Theorem 3.1 already proved. Let $C = \bigcup_{n \in N} C_n$. Consequently, $[C]_{g-seq}$ is a sequentially-g-connected subset of T by Corollary 2.10, and C is sequentially-g-connected by Theorem 2.11. In order to prove that T is a sequentially-g- connected space, it will suffice to prove that $[C]_{g-seq} = T$. For every $\alpha = (\alpha_j) \in T$ and $n \in N$, let $t_n \in T$ be the point defined by $x_{n,j} = \alpha_j$ for $j \le n$ and $x_{n,j} = \beta_j$ for $j > n$, where $x_{n,j}$ is the j-th coordinate of t_n. Therefore, $(t_n) \in T$ g-converges to α, by Theorem 3.1. Hence $[C]_{g-seq} = T$ and so T is sequentially-g-connected.

Y issequentially-g-connected.

References

1. *K. Balachandran, P. Sundaram and H. Maki (1991), On Generalized continuous functions in topological spaces, Mem. Fac. Sci. Kochi Univ., 12, 5 - 13.*

2. L. Caldas and T. Jafari (2004), On g-US spaces, Universitatea Din Bacau Studii Si Cercetari Stiintifice Seria: Matematica, 14, 13 - 20.

3. W. Dunham (1982), A new closure operator for non-T_1 topologies, Kyungpook Math. J., 22, 55 - 60.

4. W. Dunham and N. Levine (1980), Further results on generalized closed sets in topology, Kyungpook Math. J., 20, 169 - 175.

5. R. Engelking (1989), General topology (revised and completed edition), Heldermann verlag, Berlin.

6. A. Fedeli and A. Le Donne (2002), On good connected preimages, Topology Appl., 125, 489496.

7. N. Levine (1970), Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19 (2), 89 - 96.

8. V. Renukadevi and P. Vijayashanthi (2019), On I-Fréchet-Urysohn spaces and sequential –I-convergence groups, Math. Moravica, 23 (1), 119 -129. doi: 10.5937/MatMor1901119R

9. P. Vijayashanthi, V. Renukadevi and B. Prakash (2020), On countably s- Fréchet-Urysohn spaces, JCT: J. Compos. Theory, XIII (II), 969 - 976.

10. P. Vijayashanthi (2021), On sequentially-g-connected components and sequen-tially locally g-connectedness, Korean J. Math. 29 (2), 355 - 360. http://dx.doi.org/10.11568/kjm.2021.29.2.355

11. P. Vijayashanthi, and J. Kannan (2021), On Countably g-Compactness and Sequentially GO-Compactness, Korean J. Math. 29 (3), 555 - 561. http://dx.doi.org/10.11568/kjm.2021.29.3.555