ALL TREES OF DIAMETER 4 ARE ANTI-MAGIC

ABSTRACT

Except for the tree on two vertices, every tree has an anti-magic labelling, according to Hartsfield and Ringel's hypothesis. Despite the publishing of numerous study articles, the idea is still a subject of discussion. Only a small number of results concentrate on proving the anti-magic nature of particular classes of trees. The anti-magic properties of trees with a diameter of four are established in this research.

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Keywords: *diameter; anti-magic graphs; anti-magic trees*

Introduction:

In this study, we focus on straightforward, undirected, finite graphs. See the book [6] for definitions related to graph theory. A 1-1 and on-to function defined on the set of edges of a graph to 1, 2..., |E| is an anti-magic labelling of a graph G in which the vertex labels are provided as the sum of the edge labels of the edges incident to it and are distinct. Graphs are referred to as anti-magic graphs if they allow anti-magic labelling. Hartsfield and Ringel [3], who originally investigated the anti-magic labelling of graphs, proposed that every tree, with the exception of the tree on two vertices, has an anti-magic labelling.

Despite the fact that several studies on anti-magic graphs have been published, there are still many unanswered questions. For example, Liang and Zhu [5] demonstrated the anti-magic nature of 3-regular graphs, and Kaplan et al. [4] demonstrated the anti-magic nature of particular trees with certain degree requirements. Refer to Gallian's dynamic survey [2] for a complete and in-depth examination of the anti-magic graph results.

1. Main Result

We demonstrate our major finding—that tree with a diameter of four are anti-magical—in this section.

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It has been demonstrated that stars are trees of diameter 2, which is anti-magic. The anti-magic labelling of stars can be used to simply acquire the anti-magic labelling of trees with a diameter of 3. This encourages us to support our main finding—those trees with a diameter of four are anti-magical.

Theorem 2.1 Trees of diameter 4 are anti-magic.

Proof: Let T be a tree with n edges and a diameter of 4. T has a distinctive center vertex since it has a diameter of 4. (say u). Imagine the tree T now as a rooted tree with the central vertex, u, serving as the tree's root. The rooted tree has three levels, level 0, level 1, and level 2, because T has a diameter of 4. Note that the vertices in level 2 are of degree 1. Now, arrange the vertices in level 1 in such a way that degree of the vertices in level 1 are in the increasing order and as well no edges cross each other. Since trees are planar graphs, such an arrangement of vertices are always possible. Assume that there are k edges connecting the vertices in level 1 and level 2. Now, label these k edges as $1,2,3,\cdots k$. Note that the remaining n-k edges in the tree are connecting the root vertex and the vertices in the level 1. These edges can be assigned the label from the set $\{k+1, k+2, \cdots, n\}$. From our construction, it is clear that the edges labels of the edges of tree T are from the set $\{1,2,3,\cdots,k,k+1,k+2,\cdots,n\}$ and no edge labels get repeated. Hence the assignment of edge labels defined a bijective function. By our construction, if we move from the left to the right in level 2, the vertex-sum of vertices in level 2 are $1,2,3,\cdots,k$ being all the vertices in level 2 are of degree 1. Since the vertices in level 1 are arranged based on their degree in the left to right order, vertex-sum of every vertex in level 1 form a monotonically increasing order. Additionally, the root vertex's vertex-sum is higher than the vertex-sum of the vertex on level 1's extreme right. As a result, the vertex-sum of the vertices in tree T is different. So, for the given tree, we defined an anti-magic labelling. Trees with a diameter of four are hence anti-magical. Hence the evidence.

2. Illustrative example

In this section, we provide an example to illustrate our fundamental finding—all trees with a diameter of four are anti-magical. Figure 1 shows an arbitrary tree of diameter 4 with 12 edges. Figure 2 shows the root tree visualization of the tree shown in Figure 1. Figure 3 gives the anti-magic labeling of tree shown in Figure 1.

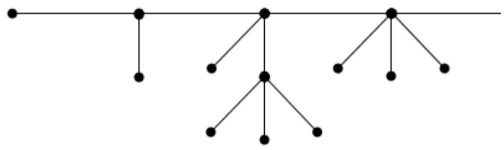

Figure 1: Diameter 4 tree with 13 edges

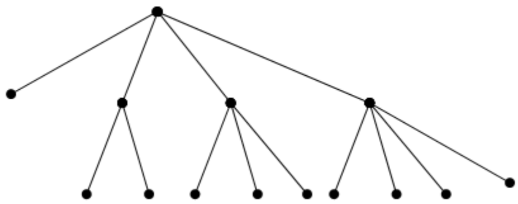

Figure 2: Rooted tree visualization of given tree

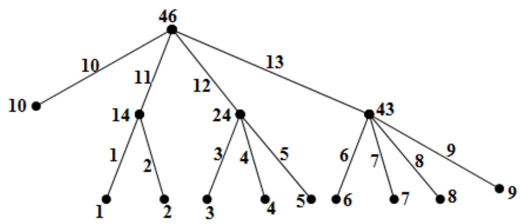

Figure 3: Anti-magic labeling of a tree with 13 edges

3. Conclusion and future directions

In this paper, we demonstrate that anti-magic labelling admits an arbitrary tree of diameter 4. We are attempting to demonstrate the anti-magic labelling of any arbitrary tree with a diameter of five or more in this direction.

In this situation, our finding is consistent with the hypothesis that all trees but the tree with two vertices are anti-magic.

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