APPLIED MATHEMATICS FOR POWER SYSTEM STATE ESTIMATION

ABSTRACT

A bewildering network like a power system network necessitates continuous monitoring and controlling to guarantee reliable operation. Constant monitoring of power systems has been done using Supervisory Control and Data Acquisition (SCADA) system for several decades. Measurements from the Remote Terminal Units (RTU) are communicated to Energy Management System (EMS). EMS processes measurement data using statistical approaches and calculates the state of the power system. Weighted Least Square (WLS) approach is most commonly used. This paper presents the application of WLS technique to different power system test systems. Test results are presented and discussed.

Keywords: *WLS, SCADA measurements, PJM system, IEEE test systems*

Introduction

With the advancements in the metering infrastructure, very accurate data (meter readings) can be obtained. Owing to the cost, the meters could not be placed at all locations in the power system network and so strategic locations for the placement of meters are identified. This problem is known as Optimal Placement Problem (OPP). The meters give real-time information about that location while the state of the rest of the locations deprived of metering infrastructure is unknown. State estimation serves as a tool to calculate the information of power system where meters are not available.The concept of State Estimation was introduced by F. Schweppe [4] – [6] in 1970. Ever since 1970, several approaches have been proposed by researchers. Recently all energy control centres have been equipped with state estimators along

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with the existing telemetry. Almost all the critical points in a power system network are equipped with Supervisory Control and Data Acquisition (SCADA) system. SCADA provides real and reactive power flows, real and reactive power injection and voltage magnitude measurements. These measurements serve as a tool for estimating the state variables of the power system network. The state variables are voltage magnitude and voltage angle at all buses [1].

Generally, state estimators are provided with measurements greater than the number of states that are to be identified. If the number of knowns (measurements) is greater than the number of unknowns (state variables), then the system is dependent and consistent. In addition, the measurements fed to the state estimator might be corrupted. Hence, statistical approaches focusing on minimising measurement error are preferred. The Weighted Least Square (WLS) method is predominantly used for state estimation.

State Estimation Concepts

Statistical approaches have been popularly used to solve state estimation problems since the beginning of the nineteenth century. It involves calculating the state of the power system network from the available redundant set of measurements. Normally, the state estimation problem is modelled as an overdetermined case [3]. The number of available measurements (N_m) is greater than the number of state variables (N_s). For an N bus system, the number of state variables, N_s is $(2N-1)$. i.e., the number of measurements fed to the state estimator, $N_m > 2N-1$.

Commonly used statistical criteria to solve a state estimation problem are Maximum Likelihood criterion, Weighted Least Square (WLS) Criterion and Maximum Variance Criterion. Out of these criteria, WLS is the most popular and promising method to solve the state estimation problem. The objective of the WLS estimator is given by:

$$min\, J(x) = \sum_{i=1}^{N_m} \frac{\left(z_i^{meas} - f_i(x)\right)^2}{{\sigma_i}^2} \qquad (2.5)$$

where $J(x)$ is called the measurement residual. This mathematical formulation applying WLS to AC and DC state estimation problems are explained by Abur and Exposito [1].

Simulation Results

This section explains DC and AC state estimation considering simple 5-bus networks.

Dc State Estimation

The network shown in Figure 2.1 is a 5-bus system. In literature, it is referred as the PJM 5-bus system. PJM is a Regional Transmission Organisation (RTO) in United States. The line data and bus data are available in the data repository [2].

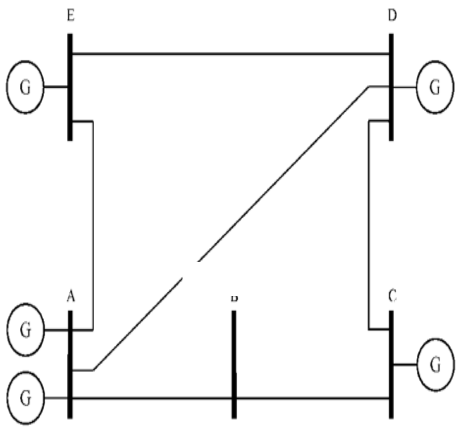

Figure 2.1 Single Line Diagram of PJM 5-Bus System

A set of 10 measurements are identified as inputs for the state estimation process. For the system shown in Figure 2.1, Number of measurements N_m = 10; Number of state variables N_s = 5; Therefore, size of the measurement Jacobian is 10×5. Following the process of DC State Estimation [1], the results obtained is given in Table 2.1. DC state estimation is a non-iterative solution.

Table 2.1 Estimated values of PJM 5-bus system using WLS state estimation

Bus	$\lvert V_{i,estimated} \rvert$	$\delta_{i,estimated}$	$P_{i,estimated}$
A	1.0000	3.2317	2.0940
B	1.0000	- 0.7937	-3.0141
C	1.0000	-0.4757	0.2344
D	1.0000	0	-3.9623
E	1.0000	4.0610	4.6481

AC State Estimation

The state estimation of the AC network shown in Figure 2.2 is explained in this subsection.

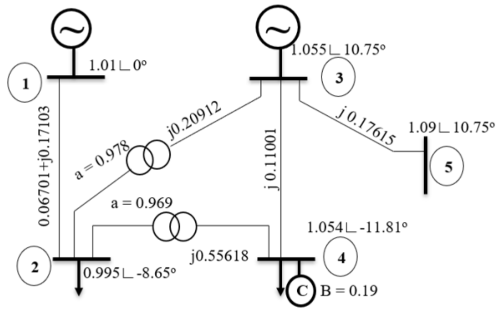

Figure 2.2 Single line diagram of 5-bus AC system

Line connecting bus 1 with bus 2 has a line charging susceptance of 0.0173 p.u. The corresponding bus admittance matrix of the network is given below:

$$Y_{bus} = \begin{bmatrix} 1.9860 - 5.0515i & -1.9860 + 5.0688i & 0 & 0 & 0 \\ -1.9860 + 5.0688i & 1.9860 - 11.9659i & 4.8895i & 0 & 1.8555i \\ 0 & 4.8895i & -19.5490i & 5.6770i & 0 \\ 0 & 0 & 5.6770i & -5.6770i & 0 \\ 0 & 1.8555i & 9.0901i & 0 & -10.6981i \end{bmatrix}$$

Power flow analysis is done using MATPOWER. A set of 12 measurements available in MATLAB package [7] are selected. A random injection of error of 3% is introduced to few measurements and is fed as input to the state estimator. Initially, a flat start assumption is made and the values of $h(x)$ are calculated. AC state estimation is an iterative process. During

every iteration, the value of $h(x)$, Jacobian matrix H and state variables are computed. The state variables are then estimated using the normal equation. Iteration proceeds till the estimated state variables converge. Table 2.2 shows the values of $h(x)$ with iteration. The estimates obtained at the end of every iteration for the considered system are organized in Table 2.3.

Table 2.2 Variation of $h(x)$ with iteration for the 5-bus AC system

Measurement Count	Initial values	At the end of iteration			
		1	2	3	4
1	1	0.0088	1.01	1.01	1.01
2	0	1	0.0037	0	0
3	0	0.1057	- 0.2096	- 0.4778	- 0.478
4	0	- 0.1555	0.7346	0.6008	0.6
5	0.152	- 0.2971	0.0444	0.0409	0.039
6	0	0.5602	0.251	0.2127	0.2134
7	0	0.1812	0.8024	0.8187	0.8187
8	0	0.0646	0.1876	0.1071	0.1072
9	0	- 0.0021	0.2608	0.1866	0.1877
10	- 0.0173	0.7965	- 0.1843	- 0.1845	- 0.1845
11	0.0594	0.1454	- 0.0499	- 0.047	- 0.0473
12	0	0.1499	- 0.0373	0.0108	0.0128

Table 2.3 Variation of estimated values with respect to iteration

Estimated state variables	At the end of iteration			
	1	2	3	4
$\|V_1\|$	1.01	1.01	1.01	1.01
$\|V_2\|$	0.9856	0.984	0.9839	0.9839
$\|V_3\|$	1.0312	1.0366	1.0369	1.0369
$\|V_4\|$	1.0837	1.0803	1.0807	1.0807
$\|V_5\|$	1.0292	1.0319	1.032	1.032
$\llcorner V_1$	0.0037	0	0	0
$\llcorner V_2$	-0.1433	-0.1497	-0.1497	-0.1497
$\llcorner V_3$	-0.2249	-0.1968	-0.1968	-0.1968
$\llcorner V_4$	-0.1108	-0.103	-0.1031	-0.1031
$\llcorner V_5$	-0.2523	-0.2157	-0.2157	-0.2157
$J(x)$	46097	1062.2	0.0985	1.57E-05

The iteration continues till the maximum deviation of state variables from one iteration to the other is less than the tolerance. Table 2.4 shows the estimated values of voltage magnitudes and voltage angles at the end of iteration. Using these estimates, the quantities in the measurement set are calculated.

Table 2.4 Estimated values of the considered 5-bus AC system using WLS state estimation

Bus No	Estimated voltage magnitude	Estimated voltage angle
1	1.01	0
2	0.995	-8.65
3	1.055	-10.74
4	1.086	-5.45
5	1.054	-11.81

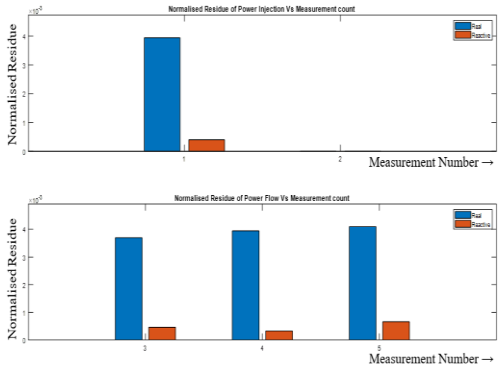

Figure 2.3 Plot of normalised residue of power measurements of the considered 5-bus AC system

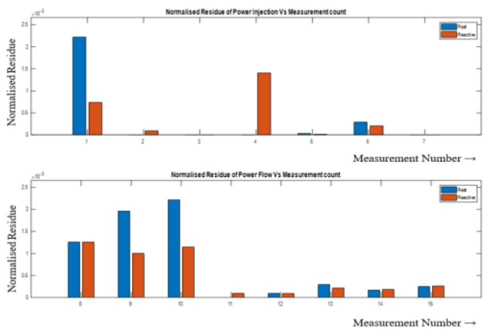

Figure 2.4 Plot of normalised residue of power measurements of IEEE 14-bus system

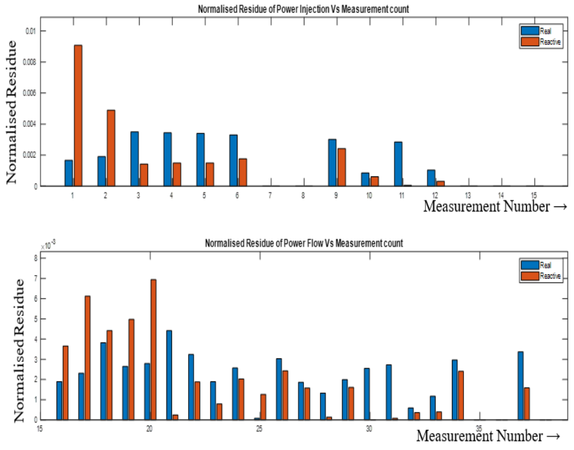

Figure 2.5 Plot of normalised residue of power measurements of IEEE 30-bus system

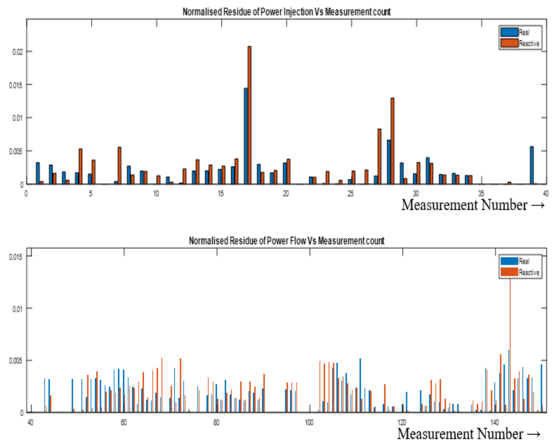

Figure 2.6 Plot of normalised residue of power measurements of IEEE 118-bus system

The normalised residue between the true and the estimated values of measurements are calculated. True values correspond to those obtained from power flow analysis. The plot of normalised residue of the power injection and flow data is shown in Figure 2.3. The algorithm is applied for IEEE 14-bus, 30-bus and 118-bus test systems.

Figure 2.4, Figure 2.5 and Figure 2.6 illustrate the plot of normalised residue of power injection and power flow measurements for IEEE 14 bus, IEEE 30 bus and IEEE 118 bus systems respectively. The normalised residues of corrupted measurements are observed to be high.

Conclusion

The basic formulation of WLS State estimation is presented considering conventional measurements. This paper discusses the application of the WLS concepts to state estimation of various test systems. It provides a comprehensive overview on the preliminaries of state estimation. This concept is fundamental of all state estimation processes and is extended to hybrid AC/DC systems as well.

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