ON $nIs_\alpha\, g$ − SUBMAXIAMAL SPACES

ABSTRACT

The intension of this paper is to instigate $nIs_\alpha g$ − submaximal space and study their characteristics and properties. Further, we have introduced $nIs_\alpha g$ − locally − ⋆ − closed set and its equivalent condition is discussed.

Keywords: *$\mathcal{M}^*$ − dense, $\mathcal{M}^*$ − codense, $nIs_\alpha g$ − locally − ⋆ − closed set, nIg − submaximal space, $nIs_\alpha g$ −submaximal space.*

1. Introduction:

M.Lellis Thivagar[4] proposed the notion of N.T.Sp. Parimala et.al[6] brought up the idea of ideals in N.T.Sp. and investigated certain properties. In this paper, we present the notion of $nIs_\alpha g$ − locally − ⋆ − closed set (briefly, $nIs_\alpha g - L^\star C.S.$) and discussed their characteristics. Further, we have introduce the notion of $nIs_\alpha g$ − submaximal space (briefly, $nIs_\alpha g$ − Sub.Max. Sp.) and investigated certain characteristics.

2.Preliminaries

Definition 2.1 A subset $\mathcal{C}$ of a N.T.Sp.$(\Gamma, \mathcal{M})$ is labeled as nano semi α − open sets (briefly, ns_α − Op.S.)[9] if there exists a $n\alpha$ − Op.S. $\mathcal{P}$ in Γ such that $\mathcal{P} \subseteq \mathcal{C} \subseteq n - cl(\mathcal{P})$ or equivalently if $\mathcal{C} \subseteq n - cl(n\alpha - int(\mathcal{P}))$.

Definition 2.2 Let $(\Gamma, \mathcal{M}, \mathcal{J})$ be a $n\mathcal{J}$ Sp. and $(.)^*_n$ be a set operator from $\mathfrak{Q}(\Gamma) \rightarrow \mathfrak{Q}(\Gamma)$, ($\mathfrak{Q}(\Gamma)$) is the powerset of Γ). For a subset $\mathfrak{A} \subset \Gamma$, $\mathfrak{A}^*_n\,(\mathcal{J}, \mathcal{M}) = \{x \in \Gamma: \mathcal{Q}_n \cap \mathfrak{A} \notin \mathcal{J}$ for every $\mathcal{Q}_n \in \mathcal{Q}_n(x)\}$ is termed as n- local function[6] of $\mathfrak{A}$ with respect to $\mathcal{J}$ and $\mathcal{N}$. We will simply write $\mathfrak{A}^*_n$ for $\mathfrak{A}^*_n(\mathcal{J}, \mathcal{N})$.

Definition 2.3 A subset $\mathbb{H}$ of a $n\mathcal{J}$ Sp. $(\Gamma, \mathcal{M}, \mathcal{J})$ is labeled as nano ideal semi α generalized Cl.S. (briefly, $nIs_\alpha g$ − Cl.S.)[10] if $\mathbb{H}^*_n \subseteq \mathfrak{K}$ whenever $\mathbb{H} \subseteq \mathfrak{K}$ and $\mathfrak{K}$ is ns_α − Op.S.

Definition 2.4 A subset $\mathbb{H}$ of a $\mathcal{J}$ Sp. $(\Gamma, \mathfrak{T}, \mathcal{J})$ is labeled as ∗ − dense set (briefly, ∗ − Dn.S.) [5] if $cl^*(\mathbb{H}) = \Gamma$.

Definition 2.5 A subset $\mathbb{H}$ of a $n\mathcal{J}$ Sp. $(\Gamma, \mathcal{M}, \mathcal{J})$ is labeled as $\mathcal{M}^*$ − Dn.S.[7] if $n - cl^*(\mathbb{H}) = \Gamma$.

G.BABY SUGANYA

Research Scholar(Reg.No : 19222072092002), Department of Mathematics, Govindammal College for Women (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012,Tamil Nadu, India), Tiruchendur, Tamil Nadu, India, sugangvs@gmail.com.

Dr. S. PASUNKILIPANDIAN

Associate Professor, Department of Mathematics, Aditanar College of Arts and Science (Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Abishekapatti, Tirunelveli-627012,Tamil Nadu, India), Tiruchendur, Tamil Nadu, India, pasunkilipandian@yahoo.com.

Dr. M.KALAISELVI

Associate Professor, Department of Mathematics, Govindammal College for Women, Tiruchendur (Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Abishekapatti, Tirunelveli-627012,Tamil Nadu, India), Tamil Nadu, India, kesriharan@gmail.com.

Definition 2.6 A subset $\mathbb{H}$ of a $n\mathcal{J}$ Sp. $(\Gamma, \mathcal{M}, \mathcal{J})$ is labeled as $pre - nI - open$ [11] if $\mathbb{H} \subseteq n - int(n - cl^*(\mathbb{H}))$.

Definition 2.7 An ideal $\mathcal{J}$ in a $n\mathcal{J}$ Sp. $(\Gamma, \mathcal{M}, \mathcal{J})$ is called $\mathcal{N} -$ condense ideal [8] if $\mathcal{N} \cap \mathcal{J} = \emptyset$.

Definition 2.8 A subset $\mathbb{H}$ of an $\mathcal{J}$ Sp. $(\Gamma, \mathcal{J})$ is labeled as $\mathcal{J}$- $L^\star C.S$.S.[5] if there exists an Op.S. Γ and a $* -$ Cl.S. $\mathfrak{f}$ such that $\mathbb{H} = \Gamma \cap \mathfrak{f}$.

Definition 2.9 A T.Sp. $(\Gamma, \mathfrak{T})$ is labeled as a $g -$ Sub.Max. Sp. [2] if every Dn.S. is $g -$ Op.S.

Definition 2.10 An $\mathcal{J}$ Sp. $(\Gamma, \mathfrak{T}, \mathcal{J})$ is labeled as an $\mathcal{J} -$ Sub.Max. Sp. [1] if every $* -$ Dn.S. is Op.S.

3. $nIs_\alpha g -$ Locally $- \star -$ Closed Sets

Definition 3.1 A subset $\mathbb{H}$ of a $n\mathcal{J}$ Sp. is $nIs_\alpha g - L^\star C.S.$ if there exists an $nIs_\alpha g -$ Op.S. $\mathcal{Q}$ and a $n^* -$ Cl.S. $\mathfrak{f}$ such that $\mathbb{H} = \mathcal{Q} \cap \mathfrak{f}$.

Example 3.2 Let $\Gamma = \{r_1, r_2, r_3, r_4\}$; $\Gamma/\mathcal{R} = \{\{r_1\}, \{r_2, r_4\}, \{r_3\}\}$; $\mathcal{X} = \{r_2, r_4\}$; $\mathcal{J} = \{\emptyset, \{r_2\}\}$. $\mathcal{M} = \{\emptyset, \Gamma, \{r_2, r_4\}\}$. $nIs_\alpha g -$Cl.S.s are $\emptyset, \Gamma, \{r_2\}, \{r_1, r_3\}, \{r_1, r_2, r_3\}, \{r_1, r_3, r_4\}$. $n^* -$ Cl.S.s are $\emptyset, \Gamma, \{r_2\}, \{r_1, r_3\}, \{r_1, r_2, r_3\}$. Here, the $nIs_\alpha g - L^\star C.S$.s are $\emptyset, \Gamma, \{r_2\}, \{r_4\}, \{r_1, r_3\}, \{r_2, r_4\}, \{r_1, r_2, r_3\}, \{r_1, r_3, r_4\}$.

Theorem 3.3 Let $(\Gamma, \mathcal{M}, \mathcal{J})$ be a $n\mathcal{J}$ Sp. and $\mathbb{H}$ be a subset of Γ. Then the underneath affirmations are analogous.

(a) $\mathbb{H}$ is $nIs_\alpha g - L^\star C.S.$

(b) $\mathbb{H} = \mathcal{Q} \cap (n - cl^*(\mathbb{H}))$ for some $nIs_\alpha g -$ Op.S. $\mathcal{Q}$.

(c) $(n - cl^*(\mathbb{H})) - \mathbb{H} = \mathbb{H}_n^* - \mathbb{H}$ is $nIs_\alpha g -$ Cl.S.

(d) $\mathbb{H} \cup (\Gamma - (n - cl^*(\mathbb{H}))) = \mathbb{H} \cup (\Gamma - \mathbb{H}_n^*)$ is $nIs_\alpha g -$ Op.S.

Proof: $(a) \Longrightarrow (b)$: If $\mathbb{H}$ is $nIs_\alpha g - L^\star C.S.$, then there exists a $nIs_\alpha g -$ Op.S. $\mathcal{Q}$ and a $n^* -$ Cl.S. $\mathfrak{f}$ such that $\mathbb{H} = \mathcal{Q} \cap \mathfrak{f}$. Clearly, $\mathbb{H} \subset \mathcal{Q} \cap n - cl^*(\mathbb{H})$. Since $\mathfrak{f}$ is $n^* -$ Cl.S., $n - cl^*(\mathbb{H}) \subset n - cl^*(\mathfrak{f}) = \mathfrak{f}$ and so $\mathcal{Q} \cap (n - cl^*(\mathbb{H})) \subset \mathcal{Q} \cap \mathfrak{f} = \mathbb{H}$. Therefore, $\mathbb{H} = \mathcal{Q} \cap (n - cl^*(\mathbb{H}))$.
$(b) \Longrightarrow (c)$: Now, $(n - cl^*(\mathbb{H})) - \mathbb{H} = \mathbb{H}_n^* - \mathbb{H} = \mathbb{H}_n^* \cap (\Gamma - \mathbb{H}) = \mathbb{H}_n^* \cap \left(\Gamma - \left(\mathcal{Q} \cap (n - cl^*(\mathbb{H}))\right)\right)$. Let $\mathfrak{K}$ be a $n -$ Op.S. such that $(n - cl^*(\mathbb{H})) - \mathbb{H} \subset \mathfrak{K}$. Then $\mathbb{H}_n^* \cap (\Gamma - \mathcal{Q}) \subset \mathbb{H}$ and so $(\Gamma - \mathcal{Q}) \subset (\Gamma - \mathbb{H}_n^*) \cup \mathfrak{K}$. Since $\Gamma - \mathcal{Q}$ is $nIs_\alpha g -$ Cl.S. and $(\Gamma - \mathbb{H}_n^*) \cup \mathfrak{K}$ is $n -$ Op.S., $n - cl^*(\Gamma - \mathcal{Q}) \subset (\Gamma - \mathbb{H}_n^*) \cup \mathfrak{K}$ and $\mathbb{H}_n^* \cap (n - cl^*(\Gamma - \mathcal{Q})) \subset \mathfrak{K}$. Since $\mathbb{H}_n^* \cap (\Gamma - \mathcal{Q}) \subset \mathbb{H}_n^*$, $\left(\mathbb{H}_n^* \cap (\Gamma - \mathcal{Q})\right)_n^* \subset (\mathbb{H}_n^*)_n^*$. Also, $\mathbb{H}_n^* \cap (\Gamma - \mathcal{Q}) \subset \Gamma - \mathcal{Q}$ implies that $\left(\mathbb{H}_n^* \cap (\Gamma - \mathcal{Q})\right)_n^* \subset (\Gamma - \mathcal{Q})_n^* \subset n - cl^*(\Gamma - \mathcal{Q})$. Therefore, $\left(\mathbb{H}_n^* \cap (\Gamma - \mathcal{Q})\right)_n^* \subset \mathbb{H}_n^* \cap (n - cl^*(\Gamma - \mathcal{Q})) \subset \mathfrak{K}$. Hence, $((n - cl^*(\mathbb{H})) - \mathbb{H})_n^* \subset \mathfrak{K}$ and so($n - cl^*(\mathbb{H})) - \mathbb{H}$ is $nIs_\alpha g -$ Cl.S.
$(c) \Longrightarrow (d)$: Since $\Gamma - ((n - cl^*(\mathbb{H})) - \mathbb{H}) = \mathbb{H} \cup (\Gamma - (n - cl^*(\mathbb{H})))$, $\mathbb{H} \cup (\Gamma - (n - cl^*(\mathbb{H})))$ is $nIs_\alpha g -$ Op.S.
$(d) \Longrightarrow (a)$: Since $\mathbb{H} = \left(\mathbb{H} \cup \left(\Gamma - (n - cl^*(\mathbb{H}))\right)\right) \cap (n - cl^*(\mathbb{H}))$ and $n - cl^*(\mathbb{H})$ is $n^* -$ Cl.S., by hypothesis, $\mathbb{H}$ is $nIs_\alpha g - L^\star C.S.$

Remark 3.4 If $\mathfrak{f}$ is a $n -$ Op. subset of a $n\mathcal{J}$ Sp. $(\Gamma, \mathcal{M}, \mathcal{J})$, then clearly $\mathfrak{f}$ is $nIs_\alpha g - L^\star C.S.$ The reverse implication is irrational.
For instance, consider $(\Gamma, \mathcal{M}, \mathcal{J})$ as in Example 3.2. If $\mathcal{Q} = \{r_1, r_3, r_4\}$; $\mathfrak{f} = \{r_1, r_3\}$ then $\mathfrak{f}_n^* = \{r_1, r_3\}$. Clearly, $\mathfrak{f}$ is $n^* -$ Cl.S. Here, $\mathcal{Q} \cap \mathfrak{f} = \{r_1, r_3\}$ is $nIs_\alpha g - L^\star C.S.$ but $\{r_1, r_3\}$ is not $nIs_\alpha g -$ Op.S.

Theorem 3.5 Let $(\Gamma, \mathcal{M}, \mathcal{J})$ be a $n\mathcal{J}$ Sp. and $\mathbb{H}$ be a subset of Γ. If $\mathbb{H}$ is $nIs_\alpha g - L^\star C.S.$ and $nI -$ Dn.S., then $\mathbb{H}$ is $nIs_\alpha g -$ Op.S.

Proof: If $\mathbb{H}$ is $nIs_\alpha g - L^\star C.S.$, by Theorem 3.3(d), $\mathbb{H} \cup (\Gamma - (n - cl^*(\mathbb{H})))$ is $nIs_\alpha g -$ Op.S. Since $\mathbb{H}$ is $nI -$ Dn.S., then $\mathbb{H}_n^* = \Gamma$ so that $n - cl^*(\mathbb{H}) = \Gamma$ which implies that $\mathbb{H}$ is $nIs_\alpha g -$ Op.S.

Corollary 3.6 Let $(\Gamma, \mathcal{M}, \mathcal{J})$ be a $n\mathcal{J}$ Sp. and $\mathbb{H}$ be $nI -$ dense subset of Γ. Then $\mathbb{H}$ is $nIs_\alpha g - L^\star C.S.$ if and only if $\mathbb{H}$ is $nIs_\alpha g -$ Op.S.

Proof: The proof is trivial.

Corollary 3.7 Let $(\Gamma, \mathcal{M}, \mathcal{J})$ be a $n\mathcal{J}$ Sp. Then the underneath affirmations are analogous.

(a) Every subset of Γ is $nIs_\alpha g - L^\star C.S.$

(b) Every $\mathcal{M}^* -$ Dn.S. is $nIs_\alpha g -$ Op.S.

Proof: $(a) \Longrightarrow (b)$: The argument emerges from Theorem 3.3(d).

$(b) \Longrightarrow (a)$: For any subset $\mathbb{H}$ of Γ, consider $Q = \mathbb{H} \cup (\Gamma - (n - cl^*(\mathbb{H})))$. Then $n - cl^*(Q) = n - cl^*(\mathbb{H}) \cup \Big(\Gamma - \big(n - cl^*(\mathbb{H})\big)\Big) = \Gamma$ so that Q is $\mathcal{M}^* -$ Dn.S. By hypothesis, Q is $nIs_\alpha g -$ Op.S. By Theorem 3.3, $\mathbb{H}$ is $nIs_\alpha g - L^\star C.S.$

4. $nIs_\alpha g -$ Submaximal Spaces

Definition 4.1 A $n\mathcal{J}$ Sp. $(\Gamma, \mathcal{M}, \mathcal{J})$ is labeled as:

(i) $nI -$ Sub.Max. Sp. if every $\mathcal{N}^* -$ Dn.S. is $n -$ Op.S.

(ii) $nIs_\alpha g -$ Sub.Max. Sp. if every $\mathcal{N}^*$ Dn.S. is $nIs_\alpha g -$ Op.S.

Example 4.2 Let $\Gamma = \{r_1, r_2, r_3, r_4\}$; $\Gamma/\mathcal{R} = \{\{r_1\}, \{r_2, r_3\}, \{r_4\}\}$; $\mathcal{X} = \{r_1, r_3\}$; $\mathcal{M} = \{\emptyset, \Gamma, \{r_1\}, \{r_1, r_2, r_3\}, \{r_1, r_2\}\}$.

(i) Let $\mathcal{J} = \{\emptyset, \{r_1\}, \{r_3\}, \{r_4\}, \{r_1, r_3\}, \{r_1, r_4\}, \{r_3, r_4\}, \{r_1, r_3, r_4\}\}$. $\mathcal{M}^* -$ Dn.S.s are $\Gamma, \{r_1, r_2\}, \{r_1, r_2, r_3\}$. In this case, every $\mathcal{N}^* -$ Dn.S. is $n -$ Op.S. Therefore, $(\Gamma, \mathcal{M}, \mathcal{J})$ is $nI -$ Sub.Max. Sp.

(ii) Let $\mathcal{J} = \{\emptyset, \{r_2\}, \{r_3\}, \{r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}, \{r_2, r_3, r_4\}\}$. $nIs_\alpha g -$ Cl.S.s are $\emptyset, \Gamma, \{r_2\}, \{r_3\}, \{r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}, \{r_2, r_3, r_4\}$. $\mathcal{M}^* -$ Dn.S.s are $\Gamma, \{r_1\}, \{r_1, r_2\}, \{r_1, r_3\}, \{r_1, r_4\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_3, r_4\}$. In this case, every $\mathcal{M}^* -$ Dn.S. is $nIs_\alpha g -$ Op.S. Therefore, $(\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_\alpha g -$ Sub.Max. Sp.

Proposition 4.3 Every $nI -$ Sub.Max. Sp. is $nIs_\alpha g -$ Sub.Max. Sp.

Proof: Let $(\Gamma, \mathcal{M}, \mathcal{J})$ be $nI -$ Sub.Max. Sp. That is, every $\mathcal{N}^* -$ Dn.S. is $n -$ Op.S. Since every $n -$ Op.S. is $nIs_\alpha g -$ Op.S., $(\Gamma, \mathcal{M}, \mathcal{J})$ be $nIs_\alpha g -$ Sub.Max. Sp.

Reamrk 4.4 A $nIs_\alpha g -$ Sub.Max. Sp. need not be a $nI -$ Sub.Max. Sp. For instance, in the Example 4.2 (ii), $(\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_\alpha g -$ Sub.Max. Sp. but not $nI -$ Sub.Max. Sp.

Definition 4.5 A subset $\mathbb{H}$ of a $n\mathcal{J}$ Sp. $(\Gamma, \mathcal{M}, \mathcal{J})$ is labeled as $\mathcal{M}^* -$ codense if its complement $\Gamma - \mathbb{H}$ is $\mathcal{M}^* -$ Dn.S.

Example 4.6 In the Example 4.2 (i), $\Gamma, \{r_1, r_2\}, \{r_1, r_2, r_3\}$ are $\mathcal{M}^* -$ Dn.S. Therefore, their complements $\emptyset, \{r_4\}, \{r_3, r_4\}$ are $\mathcal{M}^* -$ codense.

Theorem 4.7 Let $(\Gamma, \mathcal{M}, \mathcal{J})$ be a $n\mathcal{J}$ Sp. Then the underneath affirmations are analogous.

(i) $(\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_\alpha g -$ Sub.Max. Sp.

(ii) Every $\mathcal{M}^* -$ codense subset $\mathbb{H}$ of Γx is $nIs_\alpha g -$ Cl.S.

Proof: $(i) \Longrightarrow (ii)$: Assume that $(\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_\alpha g -$ Sub.Max. Sp. Let $\mathbb{H}$ be a $\mathcal{M}^* -$ codense subset of Γ. Then its complement $\mathbb{H}^c$ is $\mathcal{M}^* -$ Dn.S. Since $(\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_\alpha g -$ Sub.Max. Sp., $\mathbb{H}^c$ is $nIs_\alpha g -$ Op.S. Therefore, $\mathbb{H}$ is $nIs_\alpha g -$ Cl.S.

$(ii) \Longrightarrow (i)$: Assume that every $\mathcal{M}^* -$ subset of Γ is $nIs_\alpha g -$ Cl.S. Let $\mathbb{H}$ be a $\mathcal{M}^* -$ dense subset of Γ. Therefore, its complement $\mathbb{H}^c$ is $\mathcal{M}^* -$ codense so that it is $nIs_\alpha g -$ Cl.S. which implies $\mathbb{H}$ is $nIs_\alpha g -$ Op.S. Hence, $(\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_\alpha g -$ Sub.Max. Sp.

Theorem 4.8 Let $(\Gamma,\mathcal{M},\mathcal{J})$ be a $n\mathcal{J}$ Sp. Then the underneath affirmations are analogous.

(i) $(\Gamma,\mathcal{M},\mathcal{J})$ is $nIs_\alpha g-$ Sub.Max. Sp.

(ii) Every $pre-nI-open$ set is $nIs_\alpha g-$ Op.S.

Proof: $(i)\Longrightarrow(ii)$: Assume that the $n\mathcal{J}$ Sp. $(\Gamma,\mathcal{M},\mathcal{J})$ is $nIs_\alpha g-$ Sub.Max. Sp. and $\mathbb{H}\subseteq\Gamma$ be $pre-nI-open$. Then $\mathbb{H}=\mathcal{Q}\cap\mathfrak{K}$, $\mathcal{Q}\in\mathcal{M}$ and $\mathfrak{K}$ is $\mathcal{M}^*-$ Dn.S. Since Γ is $nIs_\alpha g-$ Sub.Max. Sp., $\mathfrak{K}$ is $nIs_\alpha g-$ Op.S. Since the intersection of two $nIs_\alpha g-$ Op.S. is a $nIs_\alpha g-$ Op.S., $\mathbb{H}$ is $nIs_\alpha g-$ Op.S.

$(ii)\Longrightarrow(i)$: Let $\mathbb{H}$ be $\mathcal{M}^*-$ Dn.S. in $(\Gamma,\mathcal{M},\mathcal{J})$. By hupothesis, $\mathbb{H}$ is $pre-nI-open$ which implies that $\mathbb{H}$ is $nIs_\alpha g-$ Op.S. Hence, $(\Gamma,\mathcal{M},\mathcal{J})$ is $nIs_\alpha g-$ Sub.Max. Sp.

Theorem 4.9 Let $(\Gamma,\mathcal{M},\mathcal{J})$ be a $n\mathcal{J}$ Sp. Then the underneath affirmations are analogous.

(i) $(\Gamma,\mathcal{M},\mathcal{J})$ is a $nIs_\alpha g-$ Sub.Max. Sp.

(ii) For every subset $\mathbb{H}\subset\Gamma$, if $\mathbb{H}$ is not a $nIs_\alpha g-$ Op.S., then $\mathbb{H}=n-int\big(n-cl^*(\mathbb{H})\big)\neq\emptyset$.

(iii) $\zeta=\{\mathcal{Q}-\mathbb{H}:\mathcal{Q}$ is $nIs_\alpha g-$ Op.S and $n-int^*(\mathbb{H})=\emptyset\}$ where ζ is the family of all $nIs_\alpha g-$ Op.S.

Proof: $(i)\Longrightarrow(ii)$: Suppose that $\mathbb{H}-\Big(n-int\big(n-cl^*(\mathbb{H})\big)\Big)=\emptyset$. Then $\mathbb{H}\subset n-int\big(n-cl^*(\mathbb{H})\big)$ which implies $\mathbb{H}$ is $pre-nI-open$. Since Γ is $nIs_\alpha g-$ Sub.Max. Sp., $\mathbb{H}$ is $nIs_\alpha g-$ Op.S. which is a contradiction. Hence, $\mathbb{H}-\Big(n-int\big(n-cl^*(\mathbb{H})\big)\Big)\neq\emptyset$.

$(ii)\Longrightarrow(i)$: Let $\mathbb{H}$ be $pre-nI-open$. Suppose that $\mathbb{H}$ is not $nIs_\alpha g-$ Op.S. Then by hypothesis, $\mathbb{H}=n-int\big(n-cl^*(\mathbb{H})\big)\neq\emptyset$ which implies that $\mathbb{H}\not\subseteq\Big(n-int\big(n-cl^*(\mathbb{H})\big)\Big)$ which is a contradiction. Hence, $\mathbb{H}$ is $nIs_\alpha g-$ Op.S. which implies that $(\Gamma,\mathcal{M},\mathcal{J})$ is a $nIs_\alpha g-$ Sub.Max. Sp.

$(i)\Longrightarrow(iii)$: Assume that $\eta=\{\mathcal{Q}-\mathbb{H}:\mathcal{Q}$ is $nIs_\alpha g-$ Op.S and $n-int^*(\mathbb{H})=\emptyset\}$. Let $\mathfrak{K}\in\zeta$. Since $\mathfrak{K}=\mathfrak{K}-\emptyset$ and $n-int^*(\emptyset)=\emptyset$ then $\zeta\subset\eta$. Let $\mathfrak{K}\in\eta$. Then $\mathfrak{K}=\mathcal{Q}-\mathbb{H}$, where $\mathcal{Q}$ is $nIs_\alpha g-$ Op.S. and $n-int^*(\mathbb{H})=\emptyset$. Then $\mathfrak{K}=\mathcal{Q}\cap(\Gamma-\mathbb{H})$. Since $n-int^*(\mathbb{H})=\emptyset,\Gamma-(n-int^*(\mathbb{H}))=n-cl^*(\Gamma-\mathbb{H})=\Gamma$. Since Γ is $nIs_\alpha g-$ Sub.Max. Sp., $\Gamma-\mathbb{H}$ is $nIs_\alpha g-$ Sub.Max. Sp. Therefore, $\mathfrak{K}$ is $nIs_\alpha g-$ Op.S. Hence, $\eta\subset\zeta$.

$(iii)\Longrightarrow(i)$: Let $\mathbb{H}$ be a $pre-nI-open$ set. Then $\mathbb{H}=\mathfrak{K}\cap\mathcal{Q}$, where $\mathfrak{K}$ is $n-$ Op.S. and $\mathcal{Q}$ is $\mathcal{M}^*-$ Dn.S. Hence, $n-cl^*(\mathcal{Q})=\Gamma$ and so $n-int^*(\Gamma-\mathcal{Q})=\emptyset$. This implies $\mathbb{H}=\mathfrak{K}-(\Gamma-\mathcal{Q})$ and $n-int^*(\Gamma-\mathcal{Q})=\emptyset$. Since every $n-$ Op.S. is $nIs_\alpha g-$ Op.S., $\mathfrak{K}$ is $nIs_\alpha g-$ Op.S. Hence, $\mathbb{H}$ is $nIs_\alpha g-$ Op.S.

Theorem 4.10 Let $(\Gamma,\mathcal{M},\mathcal{J})$ be a $n\mathcal{J}$ Sp. Then the underneath affirmations are analogous.

(i) $(\Gamma,\mathcal{M},\mathcal{J})$ is a $nIs_\alpha g-$ Sub.Max. Sp.

(ii) $n-cl^*(\mathbb{H})-\mathbb{H}$ is $nIs_\alpha g-$ Cl.S. for every $\mathbb{H}\subset\Gamma$.

Proof: $(i)\Longrightarrow(ii)$: Let $(\Gamma,\mathcal{M},\mathcal{J})$ be a $nIs_\alpha g-$ Sub.Max. Sp. and $\mathbb{H}\subset\Gamma$. Consider $\Gamma-((n-cl^*(\mathbb{H}))-\mathbb{H})=(\Gamma-(n-cl^*(\mathbb{H})))\cup\mathbb{H}$. Then $n-cl^*\big(\Gamma-(n-cl^*(\mathbb{H})-\mathbb{H})\big)=n-cl^*\Big(\big(\Gamma-n-cl^*(\mathbb{H})\big)\cup\mathbb{H}\Big)\subset\Big(\Gamma-\big(n-cl^*(\mathbb{H})\big)\Big)\cup n-cl^*(\mathbb{H})=\Gamma$. Thus, $n-cl^*\big(\Gamma-(n-cl^*(\mathbb{H})-\mathbb{H})\big)=\Gamma$. Hence, $\Gamma-((n-cl^*(\mathbb{H}))-\mathbb{H})$ is $nIs_\alpha g-$ Op.S. which implies that $n-cl^*(\mathbb{H})-\mathbb{H}$ is $nIs_\alpha g-$ Cl.S. for every $\mathbb{H}\subset\Gamma$.

$(ii)\Longrightarrow(i)$: Suppose that (ii) holds. Let $\mathbb{H}$ be $\mathcal{M}^*-$ Dn.S. in $(\Gamma,\mathcal{M},\mathcal{J})$. Since $(n-cl^*(\mathbb{H}))-\mathbb{H}$ is $nIs_\alpha g-$ Cl.S. for every

$\mathbb{H} \subset \Gamma, \Gamma - \mathbb{H}$ is $nIs_{\alpha}g -$ Cl.S. which implies that $\mathbb{H}$ is a $nIs_{\alpha}g -$ Op.S. for every $\mathbb{H} \subset \Gamma$. Hence, $(\Gamma, \mathcal{M}, \mathcal{J})$ is a $nIs_{\alpha}g -$ Sub.Max. Sp.

References

1. *Acikgoz.A, Yuksel.S and Noiri.T (2005), $\alpha - I -$ preirresolute functions and $\beta - I -$ preirresolute functions, Bull.Malayas.Sci.Soc.(2)(28),(1),1-8.*

2. *Balachandran.K, Sundaram.P and Maki.H (1996), Generalised locally closed sets and GLC- continuous functions, Indian J. Pure and Applied Math., 27(3), 235-244.*

3. *Bahvani.K and Sivaraj.D(2015), $I_g -$ Submaximal Spaces, Bol.Soc.Paran.Mat.,Vol.33 :105-110.*

4. *Lellis Thivagar.M and Carmel Richard (2013), On nano forms of weakly open sets, International journal of mathematics and statistics invention, 1(1):31–37.*

5. *Navaneethakrishnan. M and Sivaraj.D (2009), Generalised locally closed sets in ideal topological spaces, Bull.Allahabad Math.Soc, Vol.24,Part 1, 13-19.*

6. *Parimala.M, Jafari.S and Murali.S (2017), Nano ideal generalized closed sets in nano ideal topological spaces, In Annales Univ. Sci. Budapest, volume 60, pages 3–11.*

7. *Parimala.M, Jeevitha.R, and Selvakumar.A (2017), A new type of weakly closed set in ideal topological spaces, rn, 55:7.*

8. *Parimala.M, Jafari.S (2018), On some new notions in nano ideal topological spaces, Eurasian Bulletin of Mathematics, Vol.1, No.3,85-93.*

9. *Qays Hatem Imran (2018), On nano semi alpha open sets, arXiv preprint arXiv:1801.09143.*

10. *Pasunkilipandian.S, Baby Suganya.G (2022) and Kalaiselvi.M, On Some New Notions using $nIs_{\alpha}g -$ closed sets in Nano Ideal Topological Spaces, Kala Sarovar Journal, Vol.25 No.02, April – June.*

11. *Rajasekaran.I and Nethaji.O (2018), Simple forms of nano open sets in an ideal nano topological spaces, Journal of New Theory, 24(2018), 35-43.*