DEGREE SUM AND DEGREE SUM ADJACENCY ENERGY OF SOME GRAPHS

ABSTRACT

In this paper, we compute the degree sum energy and degree sum adjacency energy of $D_2(K_{2p})$ *and* $J(K_p^p)$.

Keywords: *degree sum energy, degree sum adjacency energy, edge deleting graph 2 of* K_{2p}*, join of complete graph* $J(K_p^p)$.

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Introduction

In 1978, I.Gutman was introduced the new concept energy in graph theory. Let G be a simple, undirected graph with p vertices and q edges. Let A= [a_{ij}] be the adjacency matrix of the graph G. Then the eigen values of G are $\lambda_1, \lambda_2, \ldots, \lambda_p$, assumed in non increasing order. The eigen values are real and sum equal to zero if A is real symmetric. The energy[9] E(G) of G is defined to be the sum of the absolute values of the eigen values of the adjacency matrix of G. (ie)E(G)=$\sum_{i=1}^{p} |\lambda_i|$.

The degree sum matrix of a simple graph G is denoted as DS(G) is defined as DSM(G)=$[d_{ij}] = \begin{cases} d_i + d_j & if\ i \neq j \\ 0 & otherwise \end{cases}$ .The degree sum polynomial of G is the characterstic polynomial of DSM(G) denoted by $\psi[DSM(G):\gamma]$.Since DSM(G) is a real symmetric matrix its eigen values are $\gamma_1, \gamma_2, \ldots, \gamma_p$ can be ordered as $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_p$. Then degree sum energy[14] of a graph is denoted by E_{DS}(G) =$\sum_{i=1}^{p} |\gamma_i|$.

The degree sum adjacency matrix[17] DS_A(G) of a graph G is defined as DS_A(G)=$[ds_{ij}] = \begin{cases} d_i + d_j & if\ there\ is\ an\ edge\ between\ v_i\ and\ v_j \\ 0 & otherwise \end{cases}$. The Characterstic polynomial of DS_A(G) is defined as $P_{DS_A}(G) = \beta^p + \alpha_1\beta^{p-1} + \alpha_2\beta^{p-2} + \cdots + \alpha_p$ .As DS_A(G) matrix is a real and symmetric, its eigen values are real and can be arranged as $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_p$ .The largest eigen value of DS_A (G) is known as spectral radius of a graph G. The Energy of a Degree Sum adjacency

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matrix DS_A E(G) can be defined as the sum

of the absolute eigen values of DS_A (G). This energy is also referred to as Zagreb energy. (ie) DS_A E(G) $=\sum_{i=1}^{p} |\beta_i|$.

Let K_{2p} be a complete graph with vertices 2p, p=1,2,3...,n. We split the vertices into two equal parts and delete the edges between the splitted parts. We obtain a disconnected graph, $D_2(K_{2p})$ [7] is of order 2p and p(p-1) edges and regular of degree p-1.

Consider a pair of complete graphs K_p with vertex set v_i, i=1,2,3,...,p and u_j, j=1,2,3,...,p. $J(K_p^p)$ [7]is obtained by joining v_i and u_j of order 2p and p^2 edges and regular of degree p.

Main Result

Lemma 2.1 [3] .

Let M, N, P and Q be matrices with M invertible then $\begin{bmatrix} M & N \\ P & Q \end{bmatrix}$ = $|M||Q - PM^{-1}N|$

Lemma 2.2[3].

Let M, N, P and Q be matrices. Let S = $\begin{bmatrix} M & N \\ P & Q \end{bmatrix}$ if M and P commutes then $|S| = |MQ - PN|$.

Lemma 2.3[7]

If $A(K_p)$ is the adjacency matrix of K_p then $S_p\left(A(K_p)\right) = \begin{pmatrix} p-1 & -1 \\ 1 & p-1 \end{pmatrix}$.

Lemma 2.4[7]

If $A(K_p)$ is the adjacency matrix of K_p then $A^2(K_p) = (p-2)A(K_p) + (p-1)I_p$.

Theorem 2.5

If $D_2(K_{2p})$ is edge deleting graph 2 of K_{2p} then

$S_p(DS(D_2(K_{2p})))$ = $\begin{pmatrix} (2p-1)(2p-2) & -(2p-2) \\ 1 & 2p-1 \end{pmatrix}$ and $E_{DS}(D_2(K_{2p})) = 8p^2 - 12p + 4$.

Proof:

Let $D_2(K_{2p})$ be edge deleting graph 2 of K_{2p} with 2p vertices and p(p-1) edges and regular of degree p-1.

Let $\gamma = \{\gamma_1, \gamma_2, \dots, \gamma_{2p}\}$ be the eigen values of degree sum matrix of $D_2(K_{2p})$.

Then the degree sum matrix, $DS(D_2(K_{2p}))$ =

$$\begin{bmatrix} 0 & 2p-2 & 2p-2 & \dots & 2p-2 \\ 2p-2 & 0 & 2p-2 & \dots & 2p-2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2p-2 & 2p-2 & 2p-2 & \dots & 0 \end{bmatrix}$$

The characterstic polynomial of $DS(D_2(K_{2p}))$ is

$\left|\gamma I_{2p} - DS(D_2(K_{2p}))\right| = (2p-2)\left|\gamma I_{2p} - A(K_{2p})\right|$ where $A(K_{2p})$ is the adjacency matrix of complete graph with 2p vertices .

By lemma 2.3, we get

$\left|\gamma I_{2p} - DS(D_2(K_{2p}))\right| = (\gamma - (2p-2)(2p-1))(\gamma + (2p-2))^{2p-1}]$.

Therefore $S_p(DS(D_2(K_{2p}))) = \begin{pmatrix} (2p-1)(2p-2) & -(2p-2) \\ 1 & 2p-1 \end{pmatrix}$.

and $E_{DS}(D_2(K_{2p})) = 8p^2 - 12p + 4$.

Theorem 2.6

If $D_2(K_{2p})$ is edge deleting graph 2 of K_{2p} then

$$S_p(DS_A(D_2(K_{2p}))) = \begin{pmatrix} (p-1)(2p-2) & -(2p-2) \\ 2 & 2p-2 \end{pmatrix}$$

and $DS_AE(D_2(K_{2p})) = 8p^2 - 16p + 8$.

Proof:

Let $D_2(K_{2p})$ be edge deleting graph 2 of K_{2p} with order 2p and p(p-1) edges and regular of degree p-1.

Let $\beta = \{ \beta_1, \beta_2, \dots, \beta_{2p}\}$ be the eigen values of degree sum adjacency matrix of $D_2(K_{2p})$.

Then the degree sum adjacency matrix $DS_A(D_2(K_{2p})) = \begin{bmatrix} (2p-2)A(K_p) & O_p \\ O_p & (2p-2)A(K_p) \end{bmatrix}$ where $A(K_p)$ is the adjacency matrix of K_p and O_p is the $p \times p$ zero matrix .

The characterstic polynomial of $DS_A(D_2(K_{2p}))$,

$$\left|\beta I_{2p} - DS_A(D_2(K_{2p}))\right| = (2p-2)\left|(\beta I_{2p} - A(K_p))^2\right|$$

By lemma2.3,we get $\left|\beta I_{2p} - DS_A(D_2(K_{2p}))\right| = (2p-2)[(\beta - (p-1))^2(\beta + 1)^{2p-2}]$.

Therefore $S_p(DS_A(D_2(K_{2p}))) = \begin{pmatrix} (p-1)(2p-2) & -(2p-2) \\ 2 & 2p-2 \end{pmatrix}$ and

$EDS_A (D_2(K_{2p})) = 8p^2 - 16p + 8$.

Theorem 2.7

If $J(K_p^p)$ is join of complete graph then

$S_p(DS(J(K_p^p)))) = \begin{pmatrix} 2p(2p-1) & -2p \\ 1 & 2p-1 \end{pmatrix}$ and $E_{DS}(J(K_p^p)) = 8p^2 - 4p$.

Proof:

Let $J(K_p^p)$ be join of complete graph with order 2p and p^2 edges and regular of degree p.

Let $\gamma = \{\gamma_1, \gamma_2, \dots, \gamma_{2p}\}$ be the eigen values of degree sum matrix of $J(K_p^p)$.

Then the degree sum matrix,

$$DS(J(K_p^p)) = \begin{bmatrix} 0 & 2p & 2p & \dots & 2p \\ 2p & 0 & 2p & \dots & 2p \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2p & 2p & 2p & \dots & 0 \end{bmatrix}$$

The characterstic polynomial is, $\left|\gamma I_{2p} - DS(J(K_p^p))\right| = (2p)\left|\gamma I_{2p} - A(K_{2p})\right|$

By lemma 2.3, we get $\left|\gamma I_{2p} - DS(J(K_p^p))\right| = (2p) [(\gamma - (2p-1))(\gamma + 1)^{2p-1}]$.

Therefore $S_p(DS(J(K_p^p)))) = \begin{pmatrix} 2p(2p-1) & -2p \\ 1 & 2p-1 \end{pmatrix}$

and $E_{DS}(J(K_p^p)) = 8p^2 - 4p$.

Theorem 2.8

If $J(K_p^p)$ is join of complete graph then

$S_p(DS_A(J(K_p^p)))$ = $\begin{pmatrix} (p-1)(2p-2) & -(2p-2) \\ 2 & 2p-2 \end{pmatrix}$ and $DS_AE(J(K_p^p)) = 8p^2 - 16p + 8$.

Proof:

Let $J(K_p^p)$ be join of complete graph with order 2p and p^2 edges and regular of degree p.

Let $\beta = \{\beta_1, \beta_2, \dots, \beta_{2p}\}$ be the eigen values of degree sum adjacency matrix of $J(K_p^p)$.

Then the degree sum adjacency matrix $DS_A(J(K_p^p)) = \begin{bmatrix} 2pA(K_p) & I_p \\ I_p & 2pA(K_p) \end{bmatrix}$ where $A(K_p)$ is the adjacency matrix of K_p and I_p is the $p \times p$ identity matrix .

The characterstic polynomial of $DS_A(J(K_p^p))$,

$\left|\beta I_{2p} - DS_A(J(K_p^p))\right| = (2p)\left|(\beta I_p - A(K_p))^2 - {I_p}^2\right|$

$= (2p)((\beta^2 {I_p}^2 - 2\beta A(K_p) + A^2(K_p) - {I_p}^2\ )$

By lemma 2.4 we get,

$\left|\beta I_{2p} - DS_A(J(K_p^p))\right|$ = (2p) ($\beta^2 {I_p}^2 - 2\,\beta A(K_p) + (p-2)A(K_p) + (p-1)\ I_p - {I_p}^2)$

Also by lemma2.3,we get

$\left|\beta I_{2p} - DS_A(J(K_p^p))\right|$ = (2p)[($\beta)^{p-1}(\beta + 2)^{p-1}(\beta - p)(\beta - (p-2)]$.

Therefore $S_p(DS_A(J(K_p^p)))$ = $\begin{pmatrix} 0 & -4p & 2p^2 & 2p^2 - 4p \\ p-1 & p-1 & 1 & 1 \end{pmatrix}$ and

$EDS_A\ (J(K_p^p)) = 8p^2 - 8p$.

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