ON INTUITIONISTIC GENERALIZED α-CLOSED SETS AND α-OPEN SETS

ABSTRACT

The main objective of this paper is to introduce the concepts Intuitionistic generalized α-closed sets and intuitionistic generalised α-open sets in intuitionistic topological spaces. Various properties and characteristics are given. ......
Keywords: *Intuitionistic generalised α-closed Sets, α-open Sets.*

Introduction and Preliminaries

In 1996, Coker[1] introduced the idea of "intuitionistic sets." This is a discrete version of an intuitionistic fuzzy set in which every set is a crisp set. However, it has degrees of membership and non-membership, therefore this idea offers us more adaptable methods for expressing ambiguity in mathematical objects, including those in engineering areas with conventional set logic. The idea of intuitionistic topological spaces based on intuitionistic sets, which is a traditional version of the idea in intuitionistic fuzzy topological spaces, was also introduced by Coker[2] in 2000. He also looked at compactness and the fundamental characteristics of continuous functions. There are already a number of pertinent papers in the literature.

In this work, we summarise several ideas and findings from [1,2].

Definition 1.1: [1] Consider a nonempty fixed set X. An intuitionistic set (IS for short) $\tilde{S}$ is an object having the form

$\tilde{S} = < X, S_1, S_2 >$

where S_1 and S_2 are subsets of X satisfying $S_1 \cap S_2 = \phi$. The set S_1 is called the set of members of $\tilde{S}$, while S_2 is called the set of nonmembers of $\tilde{S}$.

Example 1.2: [1] Every subset S of a nonempty set X is obviously an IS having the form $<X, S, S^C>$ *where* $S^C = X - S$.

Definition 1.3: [1] Consider a nonempty set X, and let $\tilde{S} = < X, S_1, S_2>$ and $\tilde{T} = <X, T_1, T_2>$ be IS's. Then

(a) $\tilde{S} \subseteq_{[\,]} \tilde{T}$ if and only if $S_1 \subseteq T_1$;
(b) $\tilde{S} \subseteq_{\langle\rangle} \tilde{T}$ if and only if $S_2 \supseteq T_2$;
(c) $\tilde{S} \subseteq \tilde{T}$ if and only if $\tilde{S} \subseteq_{[\,]} \tilde{T}$ and $\tilde{S} \subseteq_{\langle\rangle} \tilde{T}$;
(d) $\tilde{S} = \tilde{T}$ if and only if $\tilde{S} \subseteq \tilde{T}$ and $\tilde{T} \subseteq \tilde{S}$;
(e) $\bar{\tilde{S}} = < X, S_2, S_1>$ *is called the complement of* $\tilde{S}$ *or denoted as* $\tilde{S}^C$;

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(f) $\tilde{S} \cap \tilde{T} = <X, S_1 \cap T_1, S_2 \cup T_2>$;
(g) $\tilde{S} \cup \tilde{T} = <X, S_1 \cup T_1, S_2 \cap T_2>$;
(h) $\tilde{S} - \tilde{T} = \tilde{S} \cap \overline{\tilde{T}}$;

Definition 1.4: [1] Consider a nonempty set X and $\{\tilde{S}_i : i \in J\}$ be an arbitrary family of IS's in X, where $\tilde{S}_i = <X, S_i^{(1)}, S_i^{(2)}>$. Then

(a) $\cup \tilde{S}_i = <X, \cup S_i^{(1)}, \cap S_i^{(2)}>$ and
(b) $\cap \tilde{S}_i = <X, \cap S_i^{(1)}, \cup S_i^{(2)}>$.

Definition 1.5: [1] Consider a nonempty set X. Then $\tilde{\phi} = <X, \phi, X>$ and $\tilde{X} = <X, X, \phi>$.

The fundamental properties of inclusion and complementation are as follows:

Lemma 1.6: [1] Let $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{A}_i$ be IS's in X ($i \in J$). Then

(a) $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{C} \Rightarrow \tilde{A} \subseteq \tilde{C}$
(b) $\tilde{A}_i \subseteq \tilde{B}$ for each $i \in J \Rightarrow \cup \tilde{A}_i \subseteq \tilde{B}$
(c) $\tilde{B} \subseteq \tilde{A}_i$ for each $i \in J \Rightarrow \tilde{B} \subseteq \cap \tilde{A}_i$
(d) $\overline{\cup \tilde{A}_i} = \cap \overline{\tilde{A}_i}$
(e) $\overline{\cap \tilde{A}_i} = \cup \overline{\tilde{A}_i}$
(f) $\tilde{A} \subseteq \tilde{B} \Leftrightarrow \overline{\tilde{B}} \subseteq \overline{\tilde{A}}$
(g) $\overline{(\overline{\tilde{A}})} = \tilde{A}$
(h) $\overline{\tilde{\phi}} = \tilde{X}$
(i) $\overline{\tilde{X}} = \tilde{\phi}$

Definition 1.7: [2] An intuitionistic topology τ (IT for short) on $\tilde{X}$ is a family τ of IS's in $\tilde{X}$ satisfying the following axioms:

(T_1) $\tilde{\phi}, \tilde{X} \in \tau$,
(T_2) $\tilde{G}_1 \cap \tilde{G}_2 \in \tau$ for any $\tilde{G}_1, \tilde{G}_2 \in \tau$ and
(T_3) $\cup \tilde{G}_i \in \tau$ for any arbitrary family $\{\tilde{G}_i : i \in J\} \subseteq \tau$.

The pair $(\tilde{X}, \tau)$ is called an intuitionistic topological space (ITS for short) and any IS in τ is called an intuitionistic open set (IOS for short) in $\tilde{X}$.

Definition 1.8: [2] Consider the ITS $(\tilde{X}, \tau)$ and $\tilde{S} = <X, S_1, S_2>$ be an IS in $\tilde{X}$. Then the interior and closure of $\tilde{S}$ are defined by

$cl(\tilde{S}) = \cap\{G : G$ is an ICS in X and $\tilde{S} \subseteq G\}$ and

$int(\tilde{S}) = \cup\{H : H$ is an IOS in X and $H \subseteq \tilde{S}\}$.

It is also clear that $cl(\tilde{S})$ is an ICS and $int(\tilde{S})$ is an IOS in $\tilde{X}$, and $\tilde{S}$ is an ICS in $\tilde{X}$ if and only if $cl(\tilde{S}) = \tilde{S}$; and $\tilde{S}$ is an IOS in $\tilde{X}$ if and only if $int(\tilde{S}) = \tilde{S}$.

Lemma 1.9: [2] For any IS $\tilde{S}$ in $(\tilde{X}, \tau)$, we have $cl(\overline{\tilde{S}}) = \overline{int(\tilde{S})}$ and $int(\overline{\tilde{S}}) = \overline{cl(\tilde{S})}$

Lemma 1.10: [2] Consider the ITS $(\tilde{X}, \tau)$ and $\tilde{S}$, $\tilde{T}$ be IS's in $\tilde{X}$. Then

(a) $int(\tilde{S}) \subseteq \tilde{S}$
(b) $\tilde{S} \subseteq cl(\tilde{S})$
(c) $\tilde{S} \subseteq \tilde{T} \Rightarrow int(\tilde{S}) \subseteq int(\tilde{T})$
(d) $\tilde{S} \subseteq \tilde{T} \Rightarrow cl(\tilde{S}) \subseteq cl(\tilde{T})$
(e) $int(int(S)) = int(T)$
(f) $cl(cl(\tilde{S})) = cl(\tilde{S})$
(g) $int(\tilde{S} \cap \tilde{T}) = int(\tilde{S}) \cap int(\tilde{T})$
(h) $cl(\tilde{S} \cup \tilde{T}) = cl(\tilde{S}) \cup cl(\tilde{T})$
(i) $int(\tilde{X}) = \tilde{X}$
(j) $cl(\tilde{\phi}) = \tilde{\phi}$

Definition 1.11: [5] A subset $\tilde{S}$ of an intuitionistic topological space $(\tilde{X}, \tau)$ is said to be *Intuitionistic α-open* (IαO for short) *if* $\tilde{S} \subseteq int(cl(int(\tilde{S})))$ *and α-closed* (IαC for short) *if* $cl(int(cl(\tilde{S}))) \subseteq \tilde{S}$.

The intuitionistic α-closure of $\tilde{S}$ *is the intersection of all intuitionistic α-closed sets of* $\tilde{X}$ *containing* $\tilde{S}$, *and is represented by* $I\alpha cl(\tilde{S})$.

The α- interior of $\tilde{S}$ *is the union of all intuitionistic α-open sets of* $\tilde{X}$ *contained in* $\tilde{S}$, *and is represented by* $I\alpha int(\tilde{A})$.

2. Intuitionistic Generalised α-closed sets

Here we present intuitionistic generalised α-closed sets and investigate some of their features.

Definition 2.1: Let us consider the ITS $(\tilde{X},\tau)$ and let $\tilde{S}$ be a IS in $\tilde{X}$. A set $\tilde{S}$ is called an intuitionistic generalised α-closed set if $I\alpha cl(\tilde{S}) \subseteq \tilde{U}$ whenever $\tilde{S} \subseteq \tilde{U}$ and $\tilde{U}$ is α-open in $\tilde{X}$.

The family of all intuitionistic generalised α-closed sets (IGαCS for short) of an ITS $(\tilde{X},\tau)$ is denoted by $IG\alpha C(\tilde{X})$.

Example 2.2: Consider the set $X = \{p,q,r\}$ and the family $\tau = \{\tilde{\phi}, \tilde{P}, \tilde{Q}, \tilde{R}, \tilde{X}\}$ where $\tilde{P} = <X, \{r\},\{p,q\}>$, $\tilde{Q} = <X, \{p\},\{q,r\}>$, $\tilde{R} = <X, \{p,r\},\{q\}>$. If $\tilde{U} = < X, \{p,r\}, \{\phi\}>$, then $int(\tilde{U}) = < X,\{p,r\},\{q\} >$, $cl\left(int(\tilde{U})\right) = \tilde{X}$, $int\left(cl(int(\tilde{U}))\right) = \tilde{X} \supset \tilde{U}$. Therefore $\tilde{U}$ is an IαOS in $(\tilde{X}, \tau)$. Let $\tilde{S} = < X, \{p\}, \{q,r\}>$ be an intuitionistic subset of $\tilde{U}$ in $(\tilde{X}, \tau)$. $I\alpha cl(\tilde{S}) =< X,\{p,q\},\{r\} >$ *and* $I\alpha cl(\tilde{S}) \subseteq \tilde{U}$.

Therefore, $\tilde{S}$ is an intuitionistic generalized α-closed set in $(\tilde{X}, \tau)$.

Theorem 2.3: All ICS in $(\tilde{X}, \tau)$ is IGαCS.

Proof: The proof is clear.

From Example 2.4, we can show that the converse of the above Theorem 2.3 is false.

Example 2.4: Look at the ITS $(\tilde{X}, \tau)$ in the Example 2.2. If $\tilde{U} = < X, \{p,r\}, \{\phi\}>$, then $int(\tilde{U}) = < X,\{p,r\},\{q\} >$, $cl\left(int(\tilde{U})\right) = \tilde{X}$, $int\left(cl(int(\tilde{U}))\right) = \tilde{X} \supset \tilde{U}$. Therefore $\tilde{U}$ is an IαOS in $(\tilde{X}, \tau)$. Let $\tilde{S} = < X, \{1\}, \{3\}>$ be an intuitionistic subset of $\tilde{U}$ in $(\tilde{X}, \tau)$. $I\alpha cl(\tilde{S}) =< X,\{1,2\},\{3\} >$ *and* $I\alpha cl(\tilde{S}) \subseteq \tilde{U}$. Therefore, $\tilde{S}$ is an intuitionistic generalized α-closed set in $(\tilde{X}, \tau)$, but not ICS in $(\tilde{X},\tau)$, *since*, $cl(\tilde{S}) =< X,\{p,q\},\{r\} > \neq \tilde{S}$.

Theorem 2.5: Every IαCS in $(\tilde{X}, \tau)$ is IGαCS.

Proof: The proof is clear.

From Example 2.6, we can show that the converse of the above Theorem 2.5 is false.

Example 2.6: Look at the ITS $(\tilde{X}, \tau)$ in the Example 2.2. If $\tilde{U} = < X, \{p,q\}, \{\phi\}>$, then $int(\tilde{U}) = < X,\{p\},\{q,r\} >$, $cl\left(int(\tilde{U})\right) = \tilde{X}$, $int\left(cl(int(\tilde{U}))\right) = \tilde{X} \supset \tilde{U}$. Therefore, $\tilde{U}$ is an IαOS in $(\tilde{X}, \tau)$. Let $\tilde{S} = < X,\{q\}, \{r\}>$ be an intuitionistic subset of $\widetilde{U}$ in $(\widetilde{X}, \tau)$. $I\alpha cl(\tilde{S}) = < X, \{p, q\}, \{r\} > and$ $I\alpha cl(\tilde{S}) \subseteq \widetilde{U}$. Therefore, $\tilde{S}$ is an intuitionistic generalized α-closed set in $(\widetilde{X}, \tau)$, but not IαCS in $(\widetilde{X}, \tau)$, since $cl\left(int\left(cl(\tilde{S})\right)\right) = < X, \{p, q\}, \{r\} > \nsubseteq \tilde{S}$.

Remark 2.7: If $\widetilde{A}$ and $\widetilde{B}$ are IGαCSs then $\widetilde{A} \cup \widetilde{B}$ is also an IGαCS.

Example 2.8: Look at the ITS $(\widetilde{X}, \tau)$ in the Example 2.2. If $\widetilde{U} = < X, \{p,q\}, \{\phi\} >$, then $int(\widetilde{U}) = < X, \{p, q\}, \{r\} >$, $cl\left(int(\widetilde{U})\right) = \tilde{X}, int\left(cl(int(\widetilde{U}))\right) = \widetilde{X} \supset \widetilde{U}$. Therefore $\widetilde{U}$ is an IαOS in $(\widetilde{X}, \tau)$. Then the ISs $\tilde{A} = < X, \{q\}, \{r\} >$ and $\widetilde{B} = < X, \{p\}, \{r\} >$ are IGαCSs and $\widetilde{A} \cup \widetilde{B} = < X, \{p, q\}, \{r\} >$. Therefore, $\widetilde{A} \cup \widetilde{B}$ is an IGαCS on $(\widetilde{X}, \tau)$.

Theorem 2.9: Consider the ITS $(\widetilde{X}, \tau)$ and let $\widetilde{M}$ be an IGαCS in $(\widetilde{X}, \tau)$. Suppose that $\widetilde{N}$ is an IαCS. Then $\widetilde{M} \cap \widetilde{N}$ is an IGαCS.

Example 2.10: Consider the ITS $(\widetilde{X}, \tau)$ in the Example 2.2. Let $\tilde{A} = < X, \{q\}, \{r\} >$ be an IGαCS and $\widetilde{B} = < \{q\}, \{p\} >$ be an IαCS. Then $\widetilde{A} \cap \widetilde{B} = < X, \{q\}, \{p, r\} >$. Therefore, $\widetilde{A} \cap \widetilde{B}$ is an IGαCS on $(\widetilde{X}, \tau)$.

We characterize Igα-closed sets in the following theorem.

Theorem 2.11: Consider the ITS $(\widetilde{X}, \tau)$. Let $\widetilde{A}$ be an IS. $\widetilde{A}$ is IGαCS if and only if $I\alpha cl(\widetilde{A}) - \widetilde{A}$ contains no non-empty IαCS.

Proof: Let $\widetilde{B}$ be an Iαclosed subset of $I\alpha cl(\widetilde{A}) - \widetilde{A}$.
Now $\widetilde{A} \subset \widetilde{X} - \widetilde{B}$ and since $\widetilde{A}$ is IGαCS, we have $I\alpha cl(\widetilde{A}) \subseteq \widetilde{X} - \widetilde{B}$ or $\widetilde{B} \subset \widetilde{X} - I\alpha cl(\widetilde{A})$. Therefore, $\widetilde{B} \subset I\alpha cl(\widetilde{A}) \cap \left(\widetilde{X} - I\alpha cl(\widetilde{A})\right) = \phi$
Thus $\widetilde{B}$ is empty.
Conversely, Let $\widetilde{A} \subset \widetilde{U}$ where $\widetilde{U}$ is an IαOS in X.
If $I\alpha cl(\widetilde{A})$ not contained in $\widetilde{U}$, then $I\alpha cl(\widetilde{A}) \cap \widetilde{U} = \phi$.
Now since $\left(I\alpha cl(\widetilde{A})\right) \cap (\widetilde{X} - \widetilde{U}) \subseteq I\alpha cl(\widetilde{A}) - \widetilde{A}$ and $\left(I\alpha cl(\widetilde{A})\right) \cap (\widetilde{X} - \widetilde{U})$ is a non-empty IαCS. This gives a contradiction.

Theorem 2.12: Consider the ITS $(\widetilde{X}, \tau)$. Let $\widetilde{A}$ be an IGαCS in $(\widetilde{X}, \tau)$. Then $\widetilde{A}$ is an IαCS iff $I\alpha cl(\tilde{A}) - \widetilde{A}$ is an IαCS.

Proof:

Necessary: Let $\tilde{A}$ be an IαCS. Then $I\alpha cl(\tilde{A}) - \widetilde{A} = \phi$ which is an IαCS.

Sufficiency: Let $I\alpha cl(\widetilde{A}) - \widetilde{A}$ be an IαCS and $\widetilde{A}$ be an IGαCS. Then $I\alpha cl(\widetilde{A}) - \widetilde{A}$ does not contain any non-empty IGαCS. Because $I\alpha cl(\tilde{A}) - \widetilde{A}$ is an IαCS.
$I\alpha cl(\widetilde{A}) - \widetilde{A} = \phi$. Therefore, $\widetilde{A}$ is an IαCS.

Theorem 2.13: Consider the ITS ($\widetilde{X}$, τ). Suppose that $\widetilde{B} \subseteq \widetilde{A} \subseteq \widetilde{X}$, $\widetilde{B}$ is an IGαCS relative to $\tilde{A}$ and that is $\tilde{A}$ an IGα-closed subset of $\tilde{X}$. Then $\widetilde{B}$ is an IGαCS relative to $\tilde{X}$.

Proof: Let $\widetilde{B} \subseteq \widetilde{U}$ and suppose that $\widetilde{U}$ is IαOS in $\widetilde{X}$. Then $\widetilde{B} \subseteq \widetilde{A} \cap \widetilde{U}$ and hence $I\alpha cl_{\widetilde{A}}(\widetilde{B}) \subseteq \widetilde{A} \cap \widetilde{U}$. It follows that $\widetilde{A} \cap I\alpha cl(\widetilde{B}) \subseteq \widetilde{A} \cap \widetilde{U}$ and $\widetilde{A} \subseteq \widetilde{U} \cup \widetilde{X} - I\alpha cl(\widetilde{B})$. Since, $\widetilde{A}$ *is* $IG\widetilde{A}$ αCS *in* $\widetilde{X}$, we have $I\alpha cl(\widetilde{A}) \subseteq \widetilde{U} \cup \widetilde{X} - I\alpha cl(\widetilde{B})$. Therefore, $I\alpha cl(\widetilde{B}) \subseteq I\alpha cl(\widetilde{A}) \subseteq \widetilde{U} \cup \widetilde{X} - I\alpha cl(\widetilde{B})$ and $I\alpha cl(\widetilde{B}) \subseteq \widetilde{U}$.

Theorem 2.14: Consider the ITS ($\widetilde{X}$, τ). If $\widetilde{A}$ is IGαCS and $\widetilde{A} \subseteq \widetilde{B} \subseteq I\alpha cl(\tilde{A})$, then $\widetilde{B}$ is IGαCS.

Proof: Let $\widetilde{A}$ be an IGαCS. $I\alpha cl(\widetilde{B}) - \widetilde{B} \subseteq I\alpha cl(\tilde{A}) - \widetilde{A}$ and since $I\alpha cl(\tilde{A}) - \widetilde{A}$ has no nonempty IαC subsets, neither does $I\alpha cl(\widetilde{B}) - \widetilde{B}$. Apply Theorem 2.11.

Theorem 2.15: Consider the ITS ($\widetilde{X}$, τ). and $\widetilde{A} \subseteq \widetilde{Y} \subseteq \widetilde{X}$. *If* $\widetilde{A}$ is IGαCS in $\widetilde{X}$, *then* $\widetilde{A}$ is IGαCS relative to $\widetilde{Y}$.

Proof: Let $\widetilde{A} \subseteq \widetilde{Y} \cap \widetilde{U}$ and suppose that $\widetilde{U}$ is Iα-open in $\widetilde{X}$. Then $\widetilde{A} \subseteq \widetilde{U}$. Therefore $I\alpha cl(\tilde{A}) \subseteq \widetilde{U}$. It follows that $\widetilde{X} \cap I\alpha cl(\tilde{A}) \subseteq \widetilde{Y} \cap \widetilde{U}$.

3. Intuitionistic Generalised α-open sets

Here we present intuitionistic generalised α-open sets and investigate some of their features.

Definition 3.1: Consider the ITS ($\widetilde{X}$, τ). Let $\tilde{A}$ be a IS in $\widetilde{X}$. A set $\tilde{A}$ is said to be Intuitionistic generalised α-open set if the complement $\overline{\overline{\tilde{A}}}$ is an intuitionistic generalised α-closed set in $\widetilde{X}$. The family of all intuitionistic generalised α-open sets (IGαOS for short) of an ITS ($\widetilde{X}$,τ) is denoted by IG$\alpha O(\widetilde{X})$.

Example 3.2: Let X = {p,q,r} and consider the family τ= $\{\tilde{\phi}, \tilde{P}, \tilde{Q}, \tilde{R}, \tilde{X}\}$ where $\tilde{P}$= <X, {r},{p,q}>, $\tilde{Q}$= <X, {p},{q,r}>, $\tilde{R}$= <X, {p,r},{q}>. Let $\tilde{S}$ = < X, {p}, {ϕ}> be an IS in (X, τ) and $\overline{\overline{\tilde{S}}}$= < X, {ϕ}, {p}> which is an IGαCS in $\widetilde{X}$.

Therefore $\tilde{S}$ is an intuitionistic Generalized α-open set in ($\widetilde{X}$, τ)**.**

Theorem 3.3: For any ITS ($\widetilde{X}$,τ), we have the following:

a) Every IOS in ($\widetilde{X}$, τ) is IGαOS.
b) Every IαOS in ($\widetilde{X}$, τ) is IGαOS
c) Every intuitionistic regular open set in ($\widetilde{X}$, τ) is IGαOS. But the converses are not true.

Proof: Straightforward

Example 3.4: Look at the ITS ($\widetilde{X}$, τ) in the Example 2.2. Let $\tilde{S} = < X, \{p,r\}, \{\phi\} >$ be any IS in ($\widetilde{X}$, τ). Here $\tilde{S}$ is an IgαOS in ($\widetilde{X}$, τ), but not IOS in ($\widetilde{X}$, τ).

Example 3.5: Look at the ITS ($\widetilde{X}$, τ) in the Example 2.2. Let $\tilde{S} = < X, \{r\}, \{q\} >$ be any IS in ($\widetilde{X}$, τ). Here $\tilde{S}$ is an IGαOS in ($\widetilde{X}$, τ), but not IαOS in ($\widetilde{X}$, τ). Here $\tilde{S}$ is an IGαOS in ($\widetilde{X}$, τ), but not IαOS in ($\widetilde{X}$, τ).

Example 3.6: Look at the ITS ($\widetilde{X}$, τ) in the Example 2.2. Let $\tilde{S} = < X, \{r\}, \{p\} >$ be any IS in ($\widetilde{X}$, τ). Here $\tilde{S}$ is an IGαOS in ($\widetilde{X}$, τ), but not IROS in ($\widetilde{X}$, τ).

We characterize Igα-open sets in the following theorem.

Theorem 3.7: Consider the ITS ($\widetilde{X}$, τ) and let $\tilde{A}$ be an IS in $\widetilde{X}$. $\tilde{A}$ is IGαOS iff $\widetilde{U} \subset I\alpha int(\tilde{A})$ whenever $\widetilde{U}$ is IGαCS and $\tilde{A} - \widetilde{B} = \tilde{A} \cap \overline{\widetilde{B}}$ and $\widetilde{U} \subset \tilde{A}$.

Proof: Let $\tilde{A}$ be an IGαOS.

Suppose $\widetilde{U} \subset \tilde{A}$ *and* $\widetilde{U}$ *is* IαCS. By Definition $\tilde{X} - \tilde{A}$ is an IGαCS. Also $\tilde{X} - \tilde{A}$ is contained in the Intuitionistic α open set $\tilde{X} - \widetilde{U}$.
This implies $I\alpha cl(\tilde{X} - \tilde{A}) \subset \tilde{X} - \widetilde{U}$.
Now, $I\alpha cl(\tilde{X} - \tilde{A}) = \tilde{X} - I\alpha int(\tilde{A})$.
Hence $\tilde{X} - I\alpha int(\tilde{A}) \subset \tilde{X} - \widetilde{U}$. *That is* $\widetilde{U} \subset I\alpha int(\tilde{A})$.
Conversely suppose $\widetilde{U}$ *is an* IGαCS with $\widetilde{U} \subset I\alpha int(\tilde{A})$ *whenever* $\widetilde{U} \subset \tilde{A}$, it follows that $\tilde{X} - \tilde{A} \subseteq \tilde{X} - \widetilde{U}$ *and* $\tilde{X} - I\alpha int(\tilde{A}) \subset \tilde{X} - \widetilde{U}$.
That is $I\alpha cl(\tilde{X} - \tilde{A}) \subset \tilde{X} - \widetilde{U}$.

Hence $\tilde{X} - \tilde{A}$ is IGαCS. *Therefore* $\tilde{A}$ *becomes* IGαOS.

Theorem 3.8: Consider the ITS ($\widetilde{X}$, τ). If $\tilde{A}$ and $\widetilde{B}$ are separated IGαOSs, then $\tilde{A} \cup \widetilde{B}$ *is* IGαOS in $\widetilde{X}$.

Proof: Let $\widetilde{U}$ be an IαCS of $\tilde{A} \cup \widetilde{B}$. Then $\widetilde{U} \cap I\alpha cl(\tilde{A}) \subseteq \tilde{A}$ *and hence* $\widetilde{U} \cap I\alpha cl(\tilde{A}) \subseteq I\alpha cl(\tilde{A}) \cap (\tilde{A} \cup \widetilde{B}) \subseteq \tilde{A} \cup \tilde{\phi} = \tilde{A}$.
Similarly, $\widetilde{U} \cap I\alpha cl(\widetilde{B}) \subset \widetilde{B}$.
Hence $\widetilde{U} \cap I\alpha cl(\tilde{A}) \subset I\alpha int(\tilde{A})$ *and* $\widetilde{U} \cap I\alpha cl(\widetilde{B}) \subset I\alpha int(\widetilde{B})$.

Hence $\widetilde{U} = \widetilde{U} \cap (\tilde{A} \cup \widetilde{B}) = (\widetilde{U} \cap \tilde{A}) \cup (\widetilde{U} \cap \widetilde{B})$
$\subset (\widetilde{U} \cap I\alpha cl(\tilde{A})) \cup (\widetilde{U} \cap I\alpha cl(\widetilde{B})) \subset I\alpha int(\tilde{A}) \cup I\alpha int(\widetilde{B})$
$\subset I\alpha int(\tilde{A} \cup \widetilde{B})$
Hence by Theorem 3.7, $\tilde{A} \cup \widetilde{B}$ is IGαOS.

Theorem 3.9: Consider the ITS ($\widetilde{X}$, τ). If $I\alpha int(\tilde{A}) \subset \widetilde{B} \subset \tilde{A}$ *and* $\tilde{A}$ *is an* IGαOS, then $\widetilde{B}$ *is an* IGαOS.

Proof: By hypothesis, $\overline{\tilde{A}} \subset \overline{\widetilde{B}} \subset \overline{I\alpha int(\tilde{A})}$
That is $\overline{\tilde{A}} \subseteq \overline{\widetilde{B}} \subseteq \overline{\widetilde{X} - I\alpha cl(\tilde{A})} = I\alpha cl(\overline{\tilde{A}})$
Now $\overline{\tilde{A}}$ is an IGαCS and hence by Theorem 3.7, $\widetilde{B}$ is an IGαOS.

Theorem 3.10: Let ($\widetilde{X}$,τ) be an ITS and $\tilde{A}$ be a IGαCS in $\widetilde{X}$ if and only if $I\alpha cl(\widetilde{A}) - \tilde{A}$ is an IGαOS.

Proof: If $\tilde{A}$ is IGαCS and $\widetilde{U}$ is an IαCS such that $\widetilde{U} \subseteq I\alpha cl(\tilde{A}) - \tilde{A}$ *then by Theorem 2.13,* $\widetilde{U} = \phi$ *and hence* $\widetilde{U} \subseteq I\alpha int(I\alpha cl(\tilde{A}) - \tilde{A})$.
Hence $I\alpha cl(\tilde{A}) - \tilde{A})$ *is an* IGαOS.
Conversely suppose $I\alpha cl(\tilde{A}) - \tilde{A}$ *is an* IGαOS.

Let $\tilde{A} \subset \tilde{O}$ *where* $\tilde{O}$ ϵ IGαO(X).
That is $I\alpha cl(\tilde{A}) \cap \tilde{O}^C \subseteq I\alpha cl(\tilde{A}) \cap \tilde{A}^C$
Thus $I\alpha cl(\tilde{A}) \cap \tilde{O}^C$ is an IαCS.
$I\alpha cl(\tilde{A}) \cap \tilde{A}^C = I\alpha cl(\tilde{A}) - \tilde{A}$
Therefore, by theorem 3.7, $I\alpha cl(\tilde{A}) \cap \tilde{O}^C \subset I\alpha int(I\alpha cl(\tilde{A}) - \tilde{A}) = \phi$.
Hence $I\alpha cl(\tilde{A}) \subset \tilde{O}$ and $\tilde{A}$ is an IGαCS.

Lemma 3.11: Let $(\tilde{X},\tau)$ be an ITS. A set $\tilde{A}$ is IGαOS if $\tilde{U} \subseteq I\alpha int(\tilde{A})$ whenever $\tilde{U}$ is an IαCS and $\tilde{U} \subseteq \tilde{A}$.

Theorem 3.12: Consider the ITS $(\tilde{X}, \tau)$. A set $\tilde{A}$ is IGαOS in $(\tilde{X}, \tau)$ if and only if $\tilde{U} = \tilde{X}$ whenever $\tilde{U}$ is IαOS and $I\alpha int(\tilde{A} \cup (\tilde{X} - \tilde{A}) \subseteq \tilde{U}$.

Proof: Suppose that $\tilde{U}$ is IαOS and that $I\alpha int(\tilde{A} \cup (\tilde{X} - \tilde{A})) \subseteq \tilde{U}$. Now $\tilde{X} - \tilde{U} \subseteq I\alpha cl(\tilde{X} - \tilde{A}) \cap \tilde{A} = I\alpha cl(\tilde{X} - \tilde{A}) - (\tilde{X} - \tilde{A})$. Since $\tilde{X} - \tilde{U}$ is IαCS and $(\tilde{X} - \tilde{U})$is IGαCS, by Theorem 3.13 it follows that $\tilde{X} - \tilde{A} = \tilde{\phi}$ or $\tilde{X} = \tilde{U}$.

Suppose that is $\tilde{F}$ a IαCS and $\tilde{F} \subseteq \tilde{A}$. By Lemma 3.13, it is sufficient to show that $\tilde{F} \subseteq I\alpha int(\tilde{A})$. Now $I\alpha int(\tilde{A} \cup (\tilde{X} - \tilde{A})) \subseteq I\alpha int(\tilde{A} \cup (\tilde{X} - \tilde{F}))$ and hence $I\alpha int(\tilde{A} \cup (\tilde{X} - \tilde{F})) = \tilde{X}$. it follows that $\tilde{F} \subseteq I\alpha int(\tilde{A})$.

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